A chance-constrained formulation for routing and dimensioning of dynamic optical WDM networks

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Abstract

In a dynamic optical WDM network, requests to establish/release a connection between a source-destination pair are generated randomly. In such a network, the routing and dimensioning problem consists in finding a route for each source-destination pair and allocating a capacity to each link in such way as to guarantee a certain quality of service.

In this article we model the problem as a chance-constrained multi-commodity flow problem. In the case of homogeneous demand (each source-destination pair has the same probability of requesting a connection) we can reduce this problem to a deterministic integer programming model. In the case of heterogeneous demand such a reduction is not possible, so we approximate the problem using sample average approximation (SAA) techniques.

We show that our formulation for the homogeneous demand problem significantly outperforms previous formulations. We also show that a strengthened version of our SAA formulation for the heterogeneous demand problem obtains near-optimal solution in reasonable time. To our knowledge, this is the first successful attempt to solve the heterogeneous problem with non-heuristic approaches. Moreover, our results suggest that SAA techniques are viable for this problem and should be further explored.

1 Introduction

The emergence of multimedia networks (e.g. video on demand, high definition TV over Internet, telemedicine, on-line games) has fuelled an ever increasing need to transmit data at high speed. As a way of illustration, for proper operation of classical e-mail applications a transmission speed of just $64x10^3$ b/s (bits per second) are required, while a high definition television service requires $18x10^6$ b/s to offer a good quality of service to the user. With thousands of users requiring multiple services like that, the network must be able to provide bandwidths in the order of 10^{12} b/s.

Currently, such high transmission speeds can only be provided by using optical fiber and WDM (Wavelength Division Multiplexing) technology. WDM allows transmission of multiple information channels (wavelengths) using a unique optical fiber. Experimental research has demonstrated the feasibility of transmitting up to 320 channels at $114x10^9$ b/s each on the same optical fiber [31] (that is, about $35x10^{12}$ b/s per optical fiber). As a result, optical WDM networks are widely deployed as transport networks around the globe.

Most optical WDM networks are currently operated in a static manner. That is, optical information channels (eg, routes and wavelength assignments used to connect different origin-destination pairs in the network) are allocated permanently to users, even during periods when there is no data transmission. This way of operating can lead to an inefficient usage of resources resulting in over-dimensioned networks, especially when the utilization level is low [30]. As a way of overcoming this drawback of static operation, in the last decade several efforts towards dynamic operation of optical networks have been carried out by the research community and standardization bodies [5, 16, 11, 15, 13]. Dynamic operation means that information channels are allocated on demand. In this way, dynamic networks could potentially provide the same service that static networks but at decreased cost, a very attractive feature to network operators.

There have been several proposals of dynamic optical WDM network architectures. Depending on the resource reservation mechanism used, they can be classified as hop-by-hop or end-to-end dynamic network architectures. In a hop-by-hop network, resources are reserved immediately before data arrives to each link along the route. If some link in the route does not have available channels to transmit data, data is dropped as soon as arrives to that link. Examples of hop-by-hop dynamic optical WDM network architectures can be found in [12, 23, 3]. In an end-to-end dynamic network architecture, data is not released through the network until an optical channel has been reserved in every link of the route. Examples of end-to-end dynamic optical WDM network architectures can be found in [25, 11, 15]. Since end-to-end dynamic optical network architectures obtain less lost data than hop-by-hop ones [29], in this article we focus on end-to-end dynamic optical WDM architectures.

In an end-to-end dynamic optical WDM network, every time a connection request (i.e. a request to establish an optical channel from the source to the destination node) is generated, the resource allocation algorithm must find a route and an available optical channel in each link of that route. We assume a network equipped with wavelength conversion capability, that is, the resource allocation algorithm must only find a route with at least one optical channel available in each link of this route, regardless the wavelength of the channel. The algorithm in charge of finding a route is known as a routing algorithm.

When, in an end-to-end network, a connection is requested but there is not enough capacity in some link of the route assigned to the origin-destination pair, a *blockage* occurs. The performance of a routing algorithm in dynamic networks is typically measured in terms of the blocking probability. Routing algorithm A is better than routing algorithm B if it obtains a lower value of blocking probability. The blocking probability of a routing algorithm is in turn very much affected by the dimensioning of the network (that is, the number of wavelengths or capacity allocated to each link). If all network links are equipped with as many wavelengths as required in the worst case, then all routing algorithms would obtain zero blocking probability. There are three types of blocking probability: network wide blocking probability, connection blocking probability and link blocking probability [27]. In this paper we focus on link blocking probability, which corresponds to the ratio between the number of connection requests blocked because of unavailability of optical channels in a given link and the total number of connection requests whose routes use that link. Since wavelengths are costly resources, network operators aim at equipping the network with the minimum number of wavelengths per link such that the blocking obtained by the routing algorithms is under a given design parameter. To do so, the typical approach is selecting a routing algorithm first and then the network is dimensioned to guarantee a given blocking probability according to the routes determined by the routing algorithm. That is, the routing problem is solved first and the dimensioning problem second. However, since routing affects dimensioning and vice versa, a model that solve both problems jointly should be formulated to obtain an optimal solution.

In this article we address the problem of jointly finding a routing and a minimum-cost dimensioning for end-to-end dynamic optical WDM networks. We formulate this problem as a multi-commodity flow model [14, 26] with additional probabilistic constraints which represent the required blocking probability level. This type of optimization framework, where a decision should be taken under probabilistic constraints, is known as *chance-constrained programming.*

Chance constrained programming was first introduced in [9] and has been extensively studied since. For a theoretical background we may refer to [22] and the references therein. Although chance constrained programming is a very flexible modeling tool to incorporate uncertainty in optimization problems, the resulting problems are usually very hard to solve explicitly. The requirement of having to satisfy a certain constraint with high probability involves computing a multidimensional integration, which can only be performed exactly for certain distributions (e.g., multivariate normal distributions). In addition, the set of feasible solutions satisfying the chance constraint is usually non convex and therefore unsuitable for most optimization algorithms. Even the evaluation of feasibility for a given candidate solution cannot be done explicitly and one has to employ Monte-Carlo simulation to check feasibility.

Different approaches have been proposed in the recent years to deal with chance constrained problems. Among these approaches, we mention the concept of *efficient points* [10, 4] and the Bernstein approximation [20]. In this paper we apply a sample-based method known as Sample Average Approximation (SAA) [21, 18]. In a series of articles [6, 8, 7] the authors study SAA (they use the term scenario approach) and find the number of samples such that the solution of the sampled problem is feasible to the original chance constrained problem with high probability. Their results assume that the feasible set is convex, which was relaxed in [18]. An application of such methodology to portfolio selection theory can be found in [21].

In Section 2, we present some definitions and notations that will be used along this article, and we describe our chance-constrained formulation of the problem. In Section 3, we study how to solve this problem under the assumption that all connections have the same traffic load. In this case, we are able to reformulate our chance-constrained problem into a integer programming model that solve our problem explicitly. In Section 4, we study the general case where connections can have different traffic loads. In this case, previous reformulation is not possible, hence we present the corresponding SAA formulation of our chance-constrained problem. In Section 5 we discuss the implementation of these models and we present computational results of these different formulations. We focus on comparing the quality and feasibility of our SAA approximation with the optimal solution of our chance-constrained problem. Finally, in Section 6 we present our conclusions about the validity of the SAA approach to solve our multi-commodity chance-constrained problem.

2 A chance-constrained formulation

We consider network topology represented by a directed graph $G = (N, L)$ where N is the set of network nodes and L is the set of unidirectional links (optical fibers). The link capacity is measured in terms of number of wavelengths of link $l \in L$ and is denoted by w_l. Let $\mathcal{C} \subset N \times N$ be the set of connections that should be routed through the network. Each connection is associated to a source and destination node, s_c and t_c, respectively. We assume the traffic generated by source node s_c to destination node t_c is governed by an ON-OFF model [1]. For this traffic model, the source is assumed to transmit at the maximum bit rate and, therefore, in the long run the traffic load ρ_c corresponds to the fraction of time that connection c was transmitting data through the network. If the traffic load ρ_c is the same for every connection c we say that the traffic load is *homogeneous*, otherwise it is *heterogeneous*.

For source node s_c, we model the ON-OFF process as independent Bernoulli random variables $(\tilde{a}_c)_{c\in\mathcal{C}}$, where $\tilde{a}_c = 1$ means that connection c is in the ON state and thus the probability that connection c is established is equal to ρ_c. Hence, the number of established connections (referred to as active connections from now on) using a given link l is also a random variable. In this context, the blocking probability of link l is the probability that the number of active connections exceeds its capacity w_l.

Given the topology G, a set of connections $\mathcal{C}$, a traffic load ρ_c for each $c \in \mathcal{C}$, and a maximum value of the blocking probability acceptable in every link, α; the problem we aim to solve is the following: to find routes r_c for each $c \in \mathcal{C}$, and capacities w_l for each $l \in L$, such that the minimum number of wavelengths is used. Therefore, the optimization chance-constrained problem that we intend to solve can be formulated as:

$$\min \sum_{l \in L} w_l$$

s.t.

$$\sum_{l \in \delta^+(s_c)} x_l^c - \sum_{l \in \delta^-(s_c)} x_l^c = -1 \qquad \forall c \in \mathcal{C} \tag{R1}$$

$$\sum_{l \in \delta^+(t_c)} x_l^c - \sum_{l \in \delta^-(t_c)} x_l^c = 1 \qquad \forall c \in \mathcal{C} \tag{R2}$$

$$\sum_{l \in \delta^+(n)} x_l^c - \sum_{l \in \delta^-(n)} x_l^c = 0 \qquad \forall c \in \mathcal{C},\ \forall n \neq s_c, t_c \tag{R3}$$

$$\mathbb{P}\left(\sum_{c \in \mathcal{C}} \tilde{a}_c x_l^c \geq w_l\right) \leq \alpha \qquad \forall l \in L. \tag{PE}$$

Formulation 1: Chance-Constrained formulation of the problem.

where $\delta^+(n)$, $\delta^-(n)$ represent the set of links incoming and outgoing to and from node n, respectively.

This model is a multi-commodity flow problem with probabilistic constraints. In a multi-commodity flow problem, binary variables x_l^c denote if the connection (commodity) $c \in \mathcal{C}$ is routed through link $l \in L$ or not. Constraints (R1)-(R2)-(R3) are the classical flow conservation constraints for each connection $c \in \mathcal{C}$. That is, the incoming traffic to node n must be equal to the outgoing traffic from node n, except in the source and destination node of connection c. In the source node s_c the difference between incoming and outgoing flows should be equal to -1. In the destination node t_c, this difference should be equal to 1. The number of active connections, which is a random variable, is equal to $\sum_{c \in \mathcal{C}} \tilde{a}_c x_l^c$ and integer variables w_l denote the capacity installed in link $l \in L$. Hence, constraint (PE) ensures the required link blocking probability.

In the case of homogeneous traffic load, we will show in the next section how to write a deterministic equivalent integer optimization program for Formulation 1. Unfortunately, to the best of our knowledge, the same cannot be done under the more general hypothesis of heterogeneous traffic load. Thus, in such case we propose a model based on Sample Average Approximation (SAA) in order to obtain good candidate solutions for Formulation 1. The resulting SAA is a challenging mixed-integer program and we will build on the works of [17] and [19] in order to strengthen the formulation by the inclusion of valid cuts.

3 Homogeneous traffic load: a deterministic IP formulation

Homogeneous traffic load is one of the situations in which we can solve exactly our chance-constrained problem. Recall that homogeneous traffic load means that each connection $c \in \mathcal{C}$ has the same probability of being active, that is, $\rho_c = \rho\ \forall c \in \mathcal{C}$ for some value ρ. In this case, for each link $l \in L$ the term $\sum_{c \in \mathcal{C}} \tilde{a}_c x_l^c$ in equation (PE) is a binomial random variable with parameters N_l and ρ, where N_l is the number of connections routed through the link l. Therefore, we can compute explicitly the number of wavelengths w_l necessary to fulfil constraint (PE), by using the binomial quantile function, in the following formula:

$$w_l = w(N_l) = \min\{w : \sum_{i=0}^{w} \binom{N_l}{i} \rho^i (1-\rho)^{N-i} \geq 1 - \alpha\} \tag{1}$$

that is, w_l is the smallest w such that the cumulated distribution probability of a binomial random variable with parameters N_l and ρ evaluated in w is larger than one minus the required link blocking probability. Note that $w(\cdot)$ does not have an analytical formula, but it is easy to pre-compute these values for every possible value of N_l.

Hence, we pre-compute the values of $w(N_l)$ for all different values of N_l, and we include additional binary variables $y_{i,l}$ such that $y_{i,l} = 1$ if and only if $N_l = i$. Therefore, we can reformulate our original chance-constrained problem using the IP formulation described in Formulation 2.

$$\min \sum_{l \in L} \sum_{i=1}^{|\mathcal{C}|} w(i) \cdot y_{i,l}$$

$s.t.$

$$\sum_{l \in \delta^+(s_c)} x_l^c - \sum_{l \in \delta^-(s_c)} x_l^c = -1 \qquad \forall c \in \mathcal{C} \tag{R1}$$

$$\sum_{l \in \delta^+(t_c)} x_l^c - \sum_{l \in \delta^-(t_c)} x_l^c = 1 \qquad \forall c \in \mathcal{C} \tag{R2}$$

$$\sum_{l \in \delta^+(n)} x_l^c - \sum_{l \in \delta^-(n)} x_l^c = 0 \qquad \forall c \in \mathcal{C},\ \forall n \neq s_c, t_c \tag{R3}$$

$$\sum_{c \in \mathcal{C}} x_l^c = \sum_{i=1}^{|\mathcal{C}|} i \cdot y_{i,l} \qquad \forall l \in L$$

$$\sum_{i=1}^{|\mathcal{C}|} y_{i,l} = 1 \qquad \forall l \in L$$

Formulation 2: Integer Programming formulation for homogeneous traffic load.

In [28] a similar integer linear programming model was proposed for solving the same problem. However, this previous model used a binary variable for each possible route of each connection $c \in \mathcal{C}$. Therefore, only small-size networks with homogeneous traffic could be studied. On the other hand, our formulation requires $O(|V| \cdot |\mathcal{C}|)$ variables, providing a more scalable model to solve larger networks.

4 Heterogeneous traffic load: a strengthened SAA formulation

In the case of heterogeneous traffic load, that is, when different source-destination pairs have different values of traffic loads ρ_c, a formulation analogous to that of Formulation 2 cannot be easily derived because the sum of independent Bernoulli random variables with different parameters ρ_c does not have a known closed form. In this case, a deterministic equivalent of the chance-constrained problem is not available and we have to rely on approximations.

4.1 Enters the sample average approximation

Since the probabilistic constraint (PE) in Formulation 1 cannot be integrated explicitly, we replace it by its corresponding sample average as follows:

$$\mathbb{P}\left(\sum_{c \in \mathcal{C}} \tilde{a}_c x_l^c \geq w_l\right) = \mathbb{E}\left(\mathbf{1}\left[\sum_{c \in \mathcal{C}} \tilde{a}_c x_l^c \geq w_l\right]\right) \approx \frac{1}{|S|} \sum_{s \in S} \mathbf{1}\left[\sum_{c \in \mathcal{C}} a_c^s x_l^c \geq w_l\right] \leq \gamma, \tag{2}$$

where the symbol $\approx$ denotes an approximation for a fixed sample size $|S|$ (that converges to the expectation as the number of samples increases due to the Law of Large Numbers) and γ is the reliability level of the SAA problem. We note that in principle the reliability level α of the original Formulation 1 and the reliability γ can be different. Furthermore, $(a_c^s)_{c \in \mathcal{C}}$ are samples extracted from the random variables $\tilde{a}_c$ for each connection $c \in \mathcal{C}$. The choices of γ and $|S|$ are crucial for the implementation and validation of SAA. We will discuss this issue throughout the article.

Approximating the expected value by its sample average is certainly not a new idea: such approximation in the context of chance-constrained programming has been very popular in the last years. Applications of this technique appear in [2, 18], and a thorough study of it can be found in [21].

In order to formulate the SAA problem of Formulation 1, we introduce binary variables $y_{s,l}$ to mimic the indicator function in equation (2). That is, $y_{s,l} = 1$ indicates a constraint violation where the number of active connections using link l in scenario s is greater than its capacity w_l. The introduction of those auxiliary variables allows us to formulate an integer programming problem that approximates our original chance constrained problem described in Formulation 1 as follows.

$$\min \sum_{l \in L} w_l$$

$$s.t.$$

$$\sum_{l \in \delta^+(s_c)} x_l^c - \sum_{l \in \delta^-(s_c)} x_l^c = -1 \qquad \forall c \in \mathcal{C} \tag{R1}$$

$$\sum_{l \in \delta^+(t_c)} x_l^c - \sum_{l \in \delta^-(t_c)} x_l^c = 1 \qquad \forall c \in \mathcal{C} \tag{R2}$$

$$\sum_{l \in \delta^+(n)} x_l^c - \sum_{l \in \delta^-(n)} x_l^c = 0 \qquad \forall c \in \mathcal{C},\ \forall n \neq s_c, t_c \tag{R3}$$

$$\sum_{c \in \mathcal{C}} a_c^s x_l^c \leq w_l + y_{s,l} \cdot \mathcal{M} \quad \forall l \in L,\ \forall s \in S. \tag{PE1}$$

$$\frac{1}{|S|} \sum_{s \in S} y_{s,l} \leq \gamma \quad \forall l \in L \tag{PE2}$$

Formulation 3: Sample Average Approximation (SAA) formulation for heterogeneous traffic load.

Note that these constraints are linear on the variables, so the final formulation is a mixed-integer linear programming problem. Additionally, this approach does not require any independence assumption of random variables $(a_c^s)_{c \in \mathcal{C}}$. Since SAA is based on sampling, a key advantage of it is that the complexity of the problem remains unaffected by relaxing the independence hypothesis (as long as the samples can be generated according to some pre-specified dependence structure).

In the case $\gamma = 0$, Formulation 3 becomes easier to solve since auxiliary variables $y_{s,l}$ are all equal to zero and therefore unnecessary. In this case, constraint (PE2) becomes $\sum_{c \in \mathcal{C}} a_c^s x_l^c \leq w_l\ \forall l \in L,\ \forall s \in S$. However, as we will show in Section 5, imposing $\gamma = 0$ usually generates feasible solutions of poor quality if compared to solutions obtained for $\gamma > 0$. The reason is that the number of sampled constraints that can be violated ($y_{s,l} = 1$) is equal to $\lfloor \gamma |S| \rfloor$, where $\lfloor a \rfloor$ denotes the largest integer less or equal than a. Our computational experiments of Section 5 show that even if only low number of constraint violations are allowed, the improvement in terms of the objective function value is significant.

Unfortunately, the resulting SAA problem with $\gamma > 0$ is computationally difficult to solve, mainly because of the requirement of the so-called "big-$\mathcal{M}$" constants, as in constraints (PE1). These constraints are known to be computationally inefficient because they lead to a loose LP relaxation of the problem, obtaining weak lower bounds. To address this drawback, a common practice is to select the smallest possible value of $\mathcal{M}$. One of the contributions of this article is to provide a methodology to strengthen our SAA formulation (Formulation 3) to deal with this problem. This strengthened formulation allow us to obtain a small value for $\mathcal{M}$, and to generate additional constraints (cuts) that provide a tighter LP relaxation. The idea builds on the ongoing work of J. Luedtke (see [17]) and we describe it here in detail.

4.2 Strengthening the SAA formulation

Suppose that for a scenario $(a_c^{s'})_{c\in\mathcal{C}}$ the set of routes is already fixed as well as the capacity (number of wavelengths) of each network link, such that none of the links have a number of active connections higher than its capacity. For another scenario s, instead, the number of active connections routed through a given link $\hat{l}$ could exceed the given capacity. How large can be this excess? In order to bound this quantity, we formulate an auxiliary problem to find a routing and dimensioning for the network that maximize this excess:

$$
\begin{aligned}
(M^{s,s'}): \qquad & \max\left(\sum_{c\in\mathcal{C}} a_c^s x_{\hat{l}}^c\right) - w_{\hat{l}} \\
\text{s.t. } & (R1)-(R2)-(R3) \\
& \sum_{c\in\mathcal{C}} a_c^{s'} x_l^c \le w_l \quad \forall l \in L
\end{aligned}
$$

Note that the objective function of this problem only depends on the variables related to the link $\hat{l}$, so in the optimal solution, $w_{\hat{l}}$ should be equal to $\sum_{c\in\mathcal{C}} a_c^{s'} x_{\hat{l}}^c$. Hence, we can rewrite problem $(M^{s,s'})$ as

$$\max \sum_{c\in\mathcal{C}} (a_c^s - a_c^{s'}) x_{\hat{l}}^c \quad \text{s.t.} (R1)-(R2)-(R3)$$

Let us assume that graph G is 2-connected. In this case, it exists a route P_c for each connection c that avoids link $\hat{l}$, and another route $\bar{P}_c$ that includes $\hat{l}$. Therefore, the optimal solution of this problem is achieved assigning the route P_c to connection c if $a_c^s < a_c^{s'}$, and assigning route $\bar{P}_c$ if $a_c^s > a_c^{s'}$. Therefore, an optimal solution for problem $(M^{s,s'})$ on a 2-connected graph has objective value $\sum_{c:a_c^s > a_c^{s'}} (a_c^s - a_c^{s'})$. Let us call $h_s^{s'}$ this quantity. Note that even if G is not 2-connected, $h_s^{s'}$ is an upper bound for problem $(M^{s,s'})$. See also that $h_s^{s'}$ does not depend on the link under analysis, it only depends on scenarios s and s'. We use these values to strengthen the formulation in the following way:

Let us consider a fixed simulation $s \in S$, and order the values of $\{h_s^{s'} \ : \ s' \in S\}$. That is, $h_s^{(0)} \le h_s^{(1)} \le h_s^{(2)} \le h_s^{(3)} \le \ldots h_s^{(|S|)}$. Note that if we have a set of routes and the capacity for each link (x_l^c, w_l) such that for scenario (i) there is no link with a number of active connections exceeding its capacity (i.e. $y_{(i),l} = 0$) then $\sum_{c\in\mathcal{C}} a_c^s x_l^c \le w_l + h_s^{(i)}$ for all link $l \in L$ and for all simulation $s \in S$.

Since an optimal solution of Formulation 3 can omit at most $k = \lfloor \gamma |S| \rfloor$ scenarios for each link, then the following constraint is valid for our SAA formulation:

$$\sum_{c\in\mathcal{C}} a_c^s x_l^c \le w_l + h_s^{(k)}$$

Moreover, if for one of the scenarios $(0) \ldots (k-1)$ the number of active connections routed through link l is smaller than its capacity (i.e., $y_{(j),l} = 0$), we can reduce the constant in the right-hand side, obtaining the following k additional constraints

$$\sum_{c\in\mathcal{C}} a_c^s x_l^c \le w_l + h_s^{(k)} + (1 - y_{(j),l})(h_s^{(j)} - h_s^{(k)}) \quad \text{for } j = 0 \ldots k-1$$

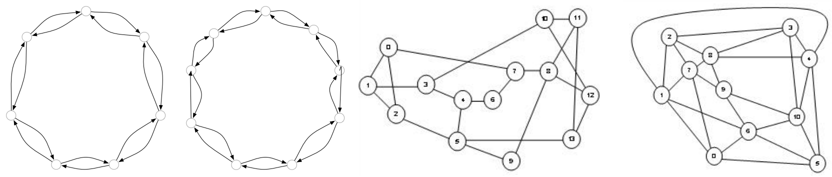
Thus, we have proven the following result:

Theorem 1 *Let be $h_s^{s'} = \sum_{c\in\mathcal{C}} (a_c^s - a_c^{s'})^+$. The following constraints are valid for Formulation 3:*

$$\sum_{c\in\mathcal{C}} a_c^s x_l^c \le w_l + h_s^{(k)} + (1 - y_{(j),l})(h_s^{(j)} - h_s^{(k)})$$

for all $j = 0 \ldots k$, where $k = \lfloor \gamma |S| \rfloor$.

Note that since $h_s^{(0)} = h_s^s = 0$, the additional constraint for $j = 0$ is exactly the constraint (PE1) with $\mathcal{M} = h_s^{(k)}$.



(a) Ring7 (b) Ring9 (c) NSFnet (d) EuroCore

Figure 1: Different topologies of optical networks.

5 Computational experiments

The main contribution of this paper is an SAA formulation of our chance-constrained problem under heterogeneous traffic load. Recall that SAA provides an approximation of our original problem, so the quality of the obtained solution should be validated. Under homogeneity we can compute the exact optimal solution of our original problem through Formulation 2. Since in the non-homogeneous case there is no explicit solution to our chance-constrained problem, we solve our SAA formulation under homogeneous traffic load, and we compare these results with the one of Formulation 2. Both situations have strong similarities, hence computational evidence that our SAA formulation works well in the homogeneous case encourages its application in the more general heterogeneous case.

5.1 Exact solutions for the homogeneous case

In the experiments we assume that all possible connections are required, that is, $\mathcal{C} = \{(s,t) \in N \times N : s \neq t\}$. First, we solve the problem for the topologies of Figure 1, under homogeneous traffic load ρ and a link blocking probability $\alpha = 10^{-2}$ and $\alpha = 10^{-6}$. The topologies studied correspond to classical examples of optical network topologies: Ring topologies (a typical topology in metropolitan networks [24]), the NSF backbone topology and a simplified version of an European network.

Table 1 and 2 show the optimal solution of this problem for different values of ρ. We refer as "IP" to the solution provided by Formulation 2. In the implementation of this problem, we use $\mathcal{M} = h_s^{(k)}$ with $k = \lfloor \gamma |S| \rfloor$, and we add additional cuts of Theorem 1 when one of these constraints is violated. For some cases, our solver was not able to compute the optimal solution after 48 hrs, so we report the best feasible solution obtained so far, and we emphasize these values in italics.

Note that if $\alpha = 0$, then the problem is deterministic and the optimal solution to the problem is obtained by routing each connection through the shortest path, and dimensioning each link with as many wavelengths as the number of connections routed through it. In Tables 1 and 2, we denote by "ShP" the solution where the routing problem is solved by assigning to each connection its shortest paths and the number of wavelengths is determined by applying equation (1).

Topology	Solution	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Ring7	IP	34	49	63	70	78	84	84	84	84
Ring7	ShP	42	56	70	70	84	84	84	84	84
Ring9	IP	63	90	117	135	153	162	177	180	180
Ring9	ShP	72	90	126	144	162	162	180	180	180
NSFnet	IP	*129*	*199*	*248*	*288*	*328*	*355*	*375*	390	390
NSFnet	ShP	144	210	256	300	336	364	382	390	390
EuroCore	IP	*77*	*113*	*136*	*157*	*165*	172	174	174	174
EuroCore	ShP	88	128	152	166	172	174	174	174	174

Table 1: Optimal solutions for different homogeneous traffic load ρ and blocking probability $\alpha = 10^{-2}$.

Topology	Solution	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Ring7	IP	68	82	84	84	84	84	84	84	84
Ring7	ShP	70	84	84	84	84	84	84	84	84
Ring9	IP	117	153	171	180	180	180	180	180	180
Ring9	ShP	126	162	180	180	180	180	180	180	180
NSFnet	IP	*259*	*332*	*368*	*386*	390	390	390	390	390
NSFnet	ShP	282	342	372	390	390	390	390	390	390
EuroCore	IP	*154*	*170*	*174*	174	174	174	174	174	174
EuroCore	ShP	166	174	174	174	174	174	174	174	174

Table 2: Optimal solutions for different homogeneous traffic load ρ and blocking probability $\alpha = 10^{-6}$.

Results are consistent with the results reported in [28]. At high traffic loads ($\rho > 0.5$) the optimal solution of the problem coincides with routing each connection through its shortest path. However, for lower values of traffic load, e.g. $\rho \leq 0.3$ there is room for improvement. As we can see, the optimal solution obtains lower network cost than the shortest path routing, specially for $\alpha = 10^{-2}$. Further analysis of the obtained solutions shows that the optimal routing obtained for low values of traffic load, attempts to concentrate several connections in a same set of links, in order to obtain lower required capacities. For more information on this phenomenon, see [28]. These results will be useful in the next section as a benchmark for the quality of the solutions found via our SAA formulation.

5.2 SAA results

Our main goal is to evaluate the quality of the solutions obtained via SAA. As mentioned in Section 4, the tractability of the approach changes dramatically if $\gamma > 0$. The resulting mixed-integer programming problems become extremely hard to solve and computationally demanding. For this reason we test our SAA formulations using the 7-node and 9-node ring topologies.

In the context of SAA problems, one key aspect is how to choose the sample size $|S|$. We considered bounds described in [18], which gives us a sample size $|S|$ such that any feasible solution for the SAA problem is also feasible for the original chance constrained Formulation 1 with arbitrarily high probability (we fixed 1-10^{-10}). We must say that the theoretical bounds available in the literature are only for problems with one chance chance-constraint and Formulation 1 has one chance constraint per link. Therefore, the estimates we will show for sample size are probably conservative. The sample sizes obtained for rings of size 7 and 9 nodes were very similar, and thus we only show the results for the 9-node ring on Table 3.

α	$\lvert S\rvert$ for Ring9, $\gamma = 0$	$\lvert S\rvert$ for Ring9, $\gamma = \alpha/2$
10^{-2}	10592	O(10^6)
10^{-6}	O(10^{14})	O(10^{14})

Table 3: Theoretical bounds on the sample size for SAA.

It can be seen that, as the reliability level increases, the minimum sample size needed to guarantee feasibility increases prohibitively. In particular, we can infer from Table 3 that an extremely high number of samples are needed for the case $\gamma = \alpha/2$. In most situations, including the problem we are considering in this article, solving problems of this size is simply impractical. We would also like to add that the number of samples needed for the case $\gamma = 0$ is significantly smaller, which encourages the use of such approximations. However, our numerical experiments show that the solutions obtained with $\gamma = 0$ are considerably worse than the ones generated by fixing $\gamma \neq 0$. This conservativeness stems from the fact that the former does not allow any scenario constraint to be violated, while the latter allows at most $\lfloor \gamma |S| \rfloor$ violations. We decided to ignore the theoretical guidelines and select the sample sizes based on computational tractability.

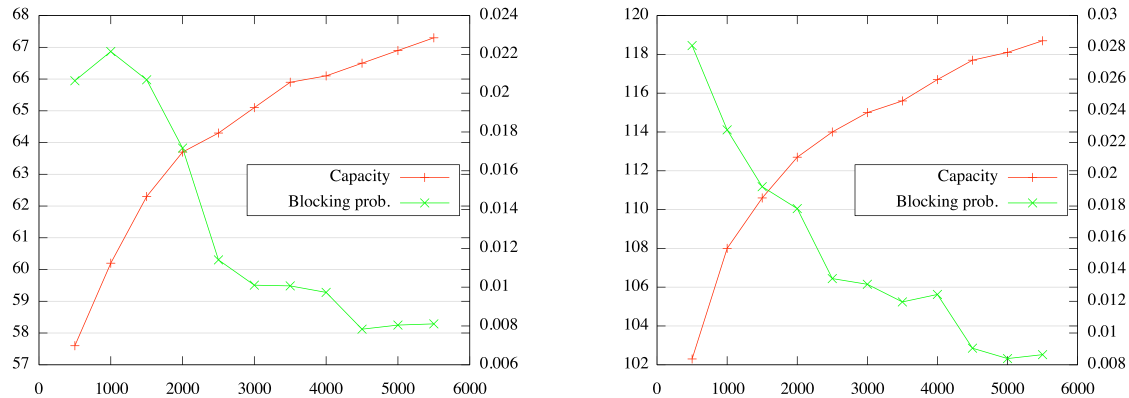


Figure 2: Resulting objective functions and blocking probabilities for Formulation 3 with $\gamma = 0$ on Ring topologies with 7 and 9 nodes with homogeneous traffic load $\rho = 0.2$ for different sample sizes.

By ignoring the theoretical guidelines, we fall into the problem of generating solutions which are not feasible for a given reliability level α. In other words, suppose we fix α and solve a single SAA problem for a sample much smaller than the ones given by Table 3. The optimal routing and the optimal capacity generated by a run of the SAA algorithm might very well be infeasible for the reliability level α. In this case we keep the optimal routing and choose the capacity for each link according to formula (1).

Figure 2 shows the results of solving Formulation 3 for $\gamma = 0$ using sample sizes between 500 and 5500, increasing by 500 samples. We solve each problem ten times, using a traffic load $\rho = 0.2$. We plot the average of the obtained objective function values (left axis) and the obtained blocking probability (right axis) of the solution. By increasing the number of simulations we naturally decrease the violation probability to the original chance-constrained problem. But this comes at a cost: since more constraints are sampled the feasible set of the SAA problem becomes smaller and therefore the quality of the solution decreases significantly. We can see that the objective function values are extremely high if compared to the true optimal values obtained (see Table 1 and Table 2). For example, the optimal solution for the 9-node ring topology at the $\alpha = 0.01$ level is 90, while the average objective function value for SAA was around 117, a 30% increase.

In Figure 3 we further investigate the quality of the solution generated by SAA with $\gamma = 0$ for different traffic loads ρ. In the vertical axis we plot the average value of the number of wavelengths required after ten runs of Formulation 3 with $\gamma = 0$ using a sample size of 500 simulations, for each $\rho = 0.1, 0.2, \ldots, 0.9$. The optimal solutions for $\alpha = 0.01$ are also shown in the figure under the label "Optimal". As we can see, for lower traffic loads the relative error between these values compared to the optimal solution can exceed 30%. Note that if we take the routing obtained in each solution, we are able to re-compute the number of wavelengths required to obtain a feasible solution for our original chance-constrained problem using equation (1). We also plot the best feasible solution obtained by this procedure among these 10 runs under the name "Feasible Wavelengths". The significant difference in the number of wavelengths required by these feasible solutions and the optimal solution indicates that the routes obtained by our SAA approximation using $\gamma = 0$ are considerably different to the ones of the optimal solution. All these results indicates that $\gamma = 0$ for this problem might not be the correct approach.

Conversely, the results obtained for $\gamma > 0$ are encouraging. We solve ten runs of our Formulation 3 with $\gamma = \alpha$ using the same 500 simulations of previous experiments, for different values of traffic loads. Due to the difficulty of solving each subproblem, we use the best integer solution obtained with the strengthened formulation after four hours of computation. In Figure 4 we plot, for different values of traffic load, the average value of objective function after ten runs. In this case, the relative error between these values and the optimal solution is considerably smaller: 2% and 3% in the worst case for the 7-node ring and 9-node

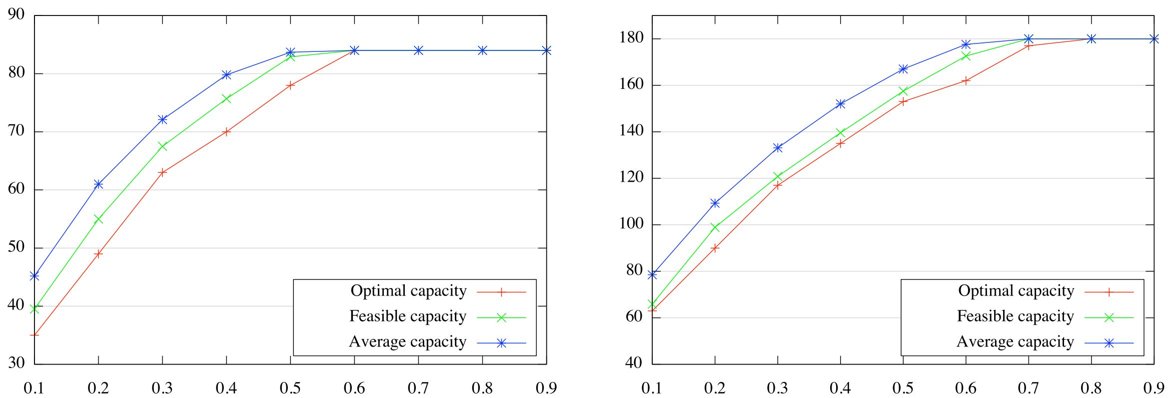


Figure 3: Resulting objective functions for Formulation 3 with $\gamma = 0$ on Ring topologies with 7 and 9 nodes for different homogeneous traffic loads. ($|S| = 500$)

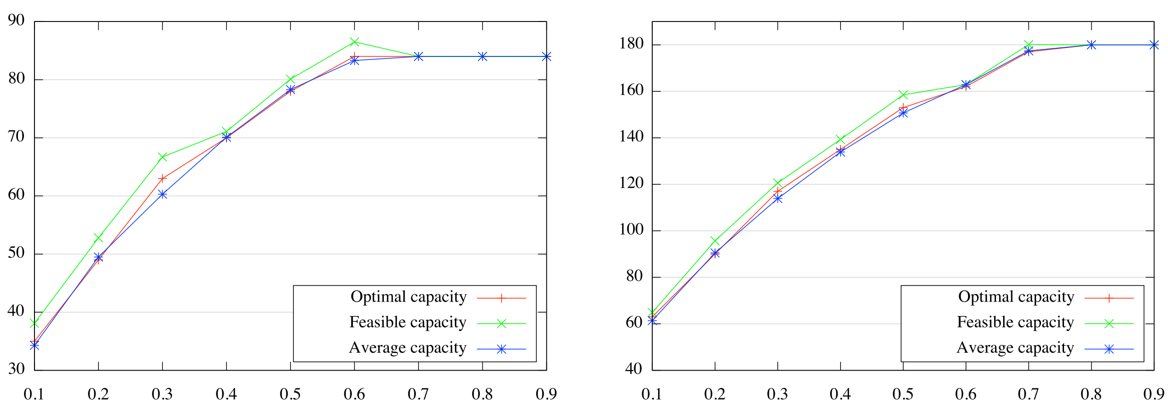


Figure 4: Resulting objective functions for Formulation 3 with $\gamma = \alpha$ on Ring topologies with 7 and 9 nodes for different homogeneous traffic loads. ($|S| = 500$)

ring, respectively. Moreover, for solutions that were infeasible with respect to the $\alpha = 0.01$ level, we also plot the best candidate obtained after forcing feasibility according to formula (1). We observe that for several values of ρ the obtained feasible solution coincides with the true optimal solution.

6 Conclusions

In this paper we solved the joint routing and dimensioning problem subject to blocking probability constraints. To do so, we formulate this problem by using chance-constrained programming.

For the homogeneous case, we propose an equivalent deterministic formulation for our chance-constrained problem that allows us to solve exactly the routing and dimensioning problem for larger networks compared with the ones reported in the literature. We implemented our formulation and solved explicitly the problem in four classic network topologies.

The importance of such closed solutions for the homogeneous case is that they served as benchmarks for experiments with the SAA method. When the approximation forbid any violation of the sampled constraints (case $\gamma = 0$) the problem could be easily solved but the quality of the solution obtained was poor. Moreover, in order to decrease the probability of violation of the original constraints we increased the sample size and observed a significant increase in the objective function value.

We performed extensive computations for the homogeneous case and showed that SAA with $\gamma = \alpha$ generated excellent candidate solutions for our original chance-constrained problem. For some values of ρ, the method was able to recover the true optimal solution obtained through Formulation 2. An important disadvantage of SAA with $\gamma = \alpha$ regards tractability: one needs to solve a nontrivial mixed-integer program in order to obtain candidate solutions. Hence, a contribution of the paper is a more efficient formulation to tackle this disadvantage, obtained by including valid cuts in SAA Formulation 3.

The choice of the sample size for SAA problems was somewhat arbitrary because the theoretical sample sizes provided by the literature were extremely large. However, given a routing it is easy to generate a dimensioning that satisfies the blocking probability with a given reliability level α. Such strategy allowed us to solve small scale SAA problems and reinforce feasibility by choosing an adequate dimensioning.

We do not have explicit solutions for the heterogeneous case so it is not possible to claim that SAA provides good approximations for this case as well. Nevertheless, the method can handle the heterogeneous case without any additional difficulty compared to the homogeneous case. Computational experiments performed in the homogeneous case suggest that SAA can generate candidates close to the optimal solution. This is very important for network operators and the telecommunication research community, because the obtained objective value provides a good benchmark to compare actual implementations of optical WDM networks, and new routing and dimensioning heuristics for this problem.

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