A Column Generation Approach for the Optimal Selection of Park-and-Ride Facilities

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1 Problem statement and solution approach

Park-and-rides (P&Rs) are facilities where users park their cars to connect to transit modes. They are usually used as incentive parkings by allowing auto users to avoid driving the most congested portion of their trips by switching to public transportation. By allowing this mode shift for the last mile of a commuters trip, it is expected that the congestion at city centers is reduced benefiting both users, and society due to a reduction of traffic congestion, reduction in energy consumption, improvements in safety, reduction of pollution, among others.

An important aspect when designing Park-and-Ride (P&R) services is the location of the P&R facility. P&Rs should be located in places where they can guarantee fast and rapid access to efficient public transit. During the planning stage, the location should also take into account the catchment area. That is, the demand that the P&R can attract. It is possible to compute the catchment area by comparing the generalized costs of the direct auto trip alternative and the generalized cost of the auto trip to the P&R plus the transfer cost and the transit trip. In that case, the cost should include not only travel, walk, wait, and transfer times but also out-of-pocket expenses (e.g., transit fares, tolls). Moreover, if the mode shares are a function of these generalized costs, the catchment area or expected demand for a P&R facility can be estimated with a Multinomial Logit (MNL) function (see for instance Holguín-Veras et al. (2012a)).

The optimal selection of P&R location can be generally defined as following. Given a demand of trips $d_{ij}$ between an origin-demand (OD) pair $(i,j) \in D$, and given a set of candidate locations $L$ for placing a P&R, we are required to select the best subset of
size $k$ that maximizes expected demand captured by the P&Rs. The demand captured is function of the ability or characteristics of each facility (i.e., total generalized cost of using this facility) to attract demand, according to a MNL function of the generalized cost. Hence, the problem can be formulated as following:

$$\max \sum_{(i,j) \in D} \sum_{l \in L} d_{ij} p_{lj}$$

(1)

$$p_{lj} = \frac{x_l \cdot e^{-\theta (c^a_{il} + c^t_{lj})}}{e^{-\theta c^a_{il}} + \sum_{h \in L} x_h \cdot e^{-\theta (c^a_{ih} + c^t_{hj})}}$$

(2)

$$\forall (i,j) \in D, \forall l \in L$$

$$\sum_{l \in L} x_l = k$$

(3)

$$x_l \in \{0, 1\}, p_{lj} \in [0, 1]$$

(4)

$$\forall (i,j) \in D, \forall l \in L$$

where $c^a_{ij}$ and $c^t_{ij}$ correspond to the generalized cost obtained by traveling from $i$ to $j$ by car and transit, respectively. Variable $x_l$ takes the value of 1 if the P&R facility located at $l$ is selected, $p_{lj}$ defines that fraction (or the probability) that the demand from $i$ to $j$ is captured by the P&R facility located in $l$. Notice that the objective function seeks to maximize the expected demand attracted as the percent of users that select P&R facility using a logit function with a parameter $\theta$ as it was proposed by Holguín-Veras et al. (2012b).

A previous formulation has been proposed by Aros-Vera et al. (2013) for this problem and it corresponds to an instance of the maximum capture problem with random utilities (Benati and Hansen, 2002). In the recent years, several mixed integer linear programming (MILP) formulations has been proposed to reformulate the non-linear formulation previously presented (see Haase and Müller (2014) for a review and a computational comparison of different formulations). However, these MILP models are still not able to real-size instance of the problem, as the case-study presented in the following section.

In this work we propose a column generation method for this problem, that will lead us to an efficient algorithm for this type of problem. Column generation is an approach commonly used in the literature for facility location problems, using an exponentially sized formulation such that each variable represents a feasible set of locations. However, in our case, instead of using variables that represent feasible subsets of $L$, we use variables associated with each possible vector of probabilities $p_{lj}$. This is possible because given a feasible subset $L' \subset L$ of locations, there is a unique vector of probabilities $p_{ij}$ for each trip $(i,j) \in D$ satisfying

$$p_{lj} = \frac{e^{-\theta (c^a_{il} + c^t_{lj})}}{e^{-\theta c^a_{il}} + \sum_{h \in L'} e^{-\theta (c^a_{ih} + c^t_{hj})}}$$

$$\forall l \in L'$$

Hence, we can define a variable $\lambda_\pi$ for each possible vector of probabilities $p$. Let $\Pi_{ij}$ the
subset of “valid” probabilities for the OD-trip \((i,j)\), and let \(\Pi^{l}_{ij}\) the subset of these vectors such that \(p^{l}_{ij} > 0\), then we can formulate the following MILP model for our problem:

\[
\max \sum_{(i,j) \in D} \sum_{\pi^{ij} \in \Pi^{l}_{ij}} w(\pi^{ij}) \cdot \lambda_{\pi^{ij}} \quad (5)
\]

\[
\sum_{\pi^{ij} \in \Pi^{l}_{ij}} \lambda_{\pi^{ij}} \geq 1, \quad \forall (i,j) \in D \quad (6)
\]

\[
\sum_{\pi^{ij} \in \Pi^{l}_{ij}} \lambda_{\pi^{ij}} \leq x_{l}, \quad \forall l \in L, \forall (i,j) \in D \quad (7)
\]

\[
\sum_{l \in L} x_{l} \leq k \quad (8)
\]

\[
\lambda_{\pi^{ij}} \geq 0, \quad x_{l} \in \{0, 1\} \quad \forall (i,j) \in D, \pi^{ij} \in \Pi^{l}_{ij}, \forall l \in L \quad (9)
\]

where \(w(\pi^{ij})\) correspond to the demand captured given the vector of probabilities \(\pi^{ij}\). Constraints (6) assure that exactly one vector of probability should be selected, and constraints (7) force that this vector should be compatible with the selected locations.

The linear relaxation of this problem is suitable to be solved by a column generation method, and to further apply a branch-and-price scheme to obtain an integer solution. However, if we relax the problem aggregating constraints (7), is it possible to solve this relaxed problem directly using a greedy algorithm, avoiding the use of a linear programming solver. This algorithm can be embedded in a branch-and-bound scheme to efficiently solve instances of large size.

2 Case study: Facility location in New York City

The data are obtained from the NYSBPM network (New York Metropolitan Transportation Council, 2009), which covers 28 counties in the states of NY, NJ and CT. The evaluation seeks to find five P&R facilities that maximize demand for destinations in Manhattan among 59 candidates (21 in Queens, 13 in Bronx, 13 in Brooklyn, and 12 in Staten Island). The data includes OD-trips between 3,269 origins with 317 destinations in Manhattan. The complete data set has been used previously by Holguín-Veras et al. (2012a). We used the demand for the morning peak (6 am to 10 am) with a total of 103,686 trips. The components of the generalized costs were derived following Holguín-Veras et al. (2012b).

We solve this instance using the previously proposed algorithm for different values of \(\theta\), requiring between 0.5 and 2.5 hours to find the optimal solution. Linear reformulations of the problem are not able to solve it due to the size of the resulting model. Solutions found are consistent with those obtained by previous studies. However, the main difference with them is that, as in the latter the evaluation has been done with each facility at a time, it does not explicitly considers competition. This is clearly shown now in our results as
at least one P&R facility is selected in each of the counties where P&Rs candidates are located, resulting in maximum coverage (see Figure 1). It can be observed that the facility selection looks reasonably valid. Note that the model starts choosing P&R facilities from the most attractive counties: those that have higher demand according to Figure 1. The P&R selection also validates Holguín-Veras et al. (2012b) theoretical findings as the best P&Rs are located close enough to the congested areas (Manhattan) to attract demand but at the outside edge of the most congested areas. This provides significant cost savings to users as all facilities selected are accessible through fast auto facilities (highways and freeways) and have strong connectivity to transit (see Figure 1).

References


