Optimal dimensioning of dynamic WDM networks

Marco Tarifeño, Alejandra Beghelli, Eduardo Moreno

***Abstract*—In this paper we formulate an integer programming model to solve the problem of dimensioning a dynamic WDM optical network taking into account the link capacity as well as the number of transmitters and receivers required in each node. Previous work has only considered the dimensioning of links. The objective function is minimizing the network cost as a function of the link capacity and the number of transmitters/receivers in each node. The constraint is given by the target blocking probability that all connections must meet. Results show that, compared to the classical approach of dimensioning just the link capacity, the proposed dimensioning method achieves a significantly lower network cost by decreasing the capacity required in the network links as well as the number of transmitters and receivers in the network nodes. In the cases studied, at low traffic loads the number of transmitters/receivers required in the network was as low as the half of the number obtained by a dimensioning that only takes the number of wavelengths into account.**

***Index Terms*—Dimensioning, integer programming, network cost, optical receivers, optical transmitters, wavelength.**

I. Introduction

In the last decade the research community and standardization bodies have made significant efforts towards dynamic operation of optical networks [1-6]. By dynamically operating a network significant benefits in terms of adaptability to traffic/topology changes and resource utilization are expected.

However, dynamic operation of a WDM network also requires expensive components such as optical switches, tunable lasers, regenerators, amplifiers and wavelength converters. Thus, it is of fundamental importance that future network designers/engineers aim at a low network cost whilst guaranteeing a good quality of service (QoS) to customers. This goal is achieved by applying efficient dimensioning methods.

Dimensioning methods proposed to date have focused on either determining the number of wavelengths per link (assuming that the number of transmistters/receivers is the maximum required; thus, blocking of connections is due only to the availability of wavelengths) [7-9] or determining the number of transmitters/receivers (keeping the number of wavelengths per link constant) [12].

In [7] two heuristic dimensioning methods were proposed. One based on simulation and the other on mathematical analysis. Both methods determine the number of wavelengths required per link such that a target blocking probability per connection is met. In a worst-case analysis, in [9] the maximum number of wavelengths required to guarantee zero blocking probability under any sequence of connection requests was determined. In [10-12] simulation and mathematical analysis were used to evaluate the impact of the number of transmitters/receivers in the network blocking. Finally, in [13] the number of wavelengths per link was assumed to be known *a priori*. Then, a time-consuming simulation-based method is proposed for determining the number of transmitters/receivers of nodes. None of these methods guaranteed an optimal solution.

In this paper, to the best of our knowledge, we propose for the first time an integer linear programming formulation to solve the joint dimensioning of transmitters/receivers per node and wavelengths per link in a dynamic WDM network. By solving the optimization problem for different cost scenarios, we find the optimal solutions to the number of wavelengths per link and the number of transmitters/receivers at each network node.

The rest of this paper is as follows: section II presents the network and traffic models; section III describes the analytic method for evaluating the blocking probability of a dynamic WDM network; in section IV the optimal dimensioning method proposed in this paper is formulated; numerical results are presented in section V and section VI concludes the paper.

II. Network and Traffic Models

We consider an end-to-end dynamic WDM network (e.g. end-to-end optical burst switching [4], optical circuit switching [14], PCE-based architectures [6]) with full wavelength conversion. This type of network has been shown to exhibit better blocking performance than hop-by-hop networks [15] or dynamic networks without wavelength conversion [7].

In such a dynamic network, once a condition (established *a priori*) for data transmission is met, a control packet is sent across the network along the specific path in order to consecutively reserve resources end-to-end. If resources cannot be reserved from source to destination, then a reject message is sent to the source node and the request is blocked. If resources are successfully reserved, an ACK message is sent

M. Tarifeño and A. Beghelli are with the Electronics Department, Universidad Técnico Federico Santa María, Valparaíso, Chile. A. Beghelli is also Honorary Associate Research, Electronic & Electrical Eng. Dept., UCL, London, UK. E. Moreno is with the Faculty of Engineering and Science, Universidad Adolfo Ibáñez, Santiago, Chile. (marco.tarifeno@postgrado.usm.cl;alejandra.beghelli@usm.cl; eduardo.moreno@uai.cl).

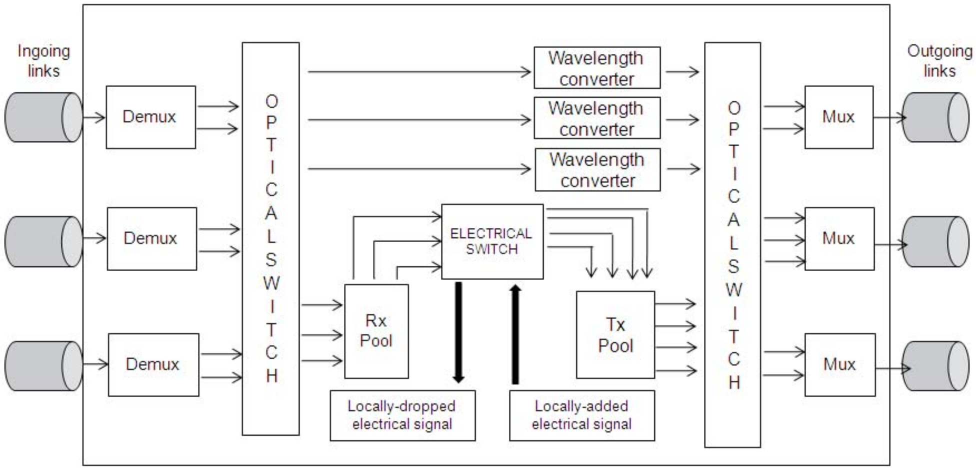

Fig. 1. Diagram of WDM node

to the source node. Upon reception of the ACK message, data transmission starts. In this paper a dynamic routing algorithm based on static routes pre-computed *a priori* (shortest path) is considered.

The network is represented by a graph $\mathcal{G} = (N, L)$, where N is the set of nodes and L is the set of unidirectional links (optical fibers). The cardinality of sets N and L is denoted by $|N|$ and $|L|$, respectively. Let C be the set of all connections. Each connection c is associated to a source-destination pair, (*s,d*). The cardinality of the set C is denoted by $|C|$ and is given by $|N|(|N|-1)$. Each connection has a fixed (static) route associated to it, denoted by $path_c$. The capacity of link $l \in L$, in terms of number of wavelengths, is denoted by W_l. The number of transmitters and receivers of node n is denoted by T_n and R_n, respectively.

Figure 1 show the architecture of WDM node considered in this paper. The Tx Pool and Rx Pool are the sets of T_n transmitters and R_n receivers, respectively. Let in_n be the set of ingoing links to node n and out_n the set of outgoing links from node $n, \forall n \in N$. In Figure 1, we can see three ingoing links with six connections; of wich three have as destination this node and three have as destination other nodes. Also we can see three outgoing links with seven connections; of wich four connections have as source this node.

We assume the traffic offered to the network by each source-destination pair governed by an ON-OFF model, suitable to represent sources of traffic in end-to-end dynamic networks. During the ON period, the source is assumed to transmit at the maximum bit rate, corresponding to the transmission rate of one wavelength. During the OFF period, the source does not transmit data. The mean duration of the ON and OFF periods are denoted by t_{ON} and t_{OFF} respectively. We assumed identical values of t_{ON} and t_{OFF} for all connections. Therefore, the traffic load offered by connection c, ρ_c, is given by:

$$\rho = \frac{t_{ON}}{t_{ON} + t_{OFF}} \quad (1)$$

III. Blocking Probabilities

The blocking probability of connection c -with source node s and destination node d- can be estimated as:

$$B_c = 1 - \left(1 - B_{s_c}^{tx}\right)\left(1 - B_{d_c}^{rx}\right) \prod_{\forall l \in path(c)} (1 - B_l) \quad (2)$$

where $B_{s_c}^{tx}$ is the blocking probability due to unavailability of transmitters in node s_c, $B_{d_c}^{rx}$ is the blocking probability due to the unavailability of receivers in node d_c and B_l is the blocking probability of link l due to unavailability of wavelengths.

To evaluate $B_{s_c}^{tx}$, $B_{d_c}^{rx}$ and B_l, the Engset formula [16] is used. In its generic form, the blocking probability B experienced by a set of y clients that arrive at rate λ (clients/time unit) at a pool of x servers that serve the clients at rate μ (clients/time unit) is given by:

$$B = \frac{\binom{y}{x}\left(\frac{\rho}{\rho-1}\right)^x}{\sum_{i=0}^{x}\binom{y}{i}\left(\frac{\rho}{\rho-1}\right)^i} \quad (3)$$

where $\rho = \lambda/\mu$ is the traffic load offered by the clients to the pool of servers.

Thus:

$$B_s^{tx} = B^{tx}(T_s) = \frac{\binom{|N|-1}{T_s}\left(\frac{\rho}{\rho-1}\right)^{T_s}}{\sum_{i=0}^{T_s}\binom{|N|-1}{i}\left(\frac{\rho}{\rho-1}\right)^i} \quad (4)$$

$$B_d^{rx} = B^{rx}(R_d) = \frac{\binom{|N|-1}{R_d}\left(\frac{\rho}{\rho-1}\right)^{R_d}}{\sum_{i=0}^{R_d}\binom{|N|-1}{i}\left(\frac{\rho}{\rho-1}\right)^i} \quad (5)$$

$$B_l = B_l(W_l) = \frac{\binom{p_l}{W_l}\left(\frac{\rho}{\rho-1}\right)^{W_l}}{\sum_{\mathrm{i}=0}^{W_l}\binom{p_l}{\mathrm{i}}\left(\frac{\rho}{\rho-1}\right)^{\mathrm{i}}} \quad (6)$$

where p_l is the number of routes using link *l*. Note that the evaluation of $B_{s_c}^{tx}$, $B_{d_c}^{rx}$ and B_l includes non-linear operations of variables. However, it only depends on the number of transmitter T_n, receivers R_n and wavelengths W_l.

IV. An Integer Linear Programming Formulation

The problem of network dimensioning, taking the number of wavelengths per link and the number of transmitters-receivers per node into account simultaneously, can be formulated as follows:

$$\min C_{net} \tag{7}$$

subject to

$$B_c \leq B_{target}, \qquad \forall c \in C \tag{8}$$

where C_{net} is the network cost, B_c is the blocking probability experienced by connection c and B_{target} is the maximum value of blocking aceptable for any connection in the network. Thus, the network must be dimensioned at minimum cost whilst guaranteeing a given level of QoS.

In this paper, the network cost (C_{net}) is assumed to be affected by the number of wavelengths required per link (as this number impacts the cost of switches and amplifiers) and the number of transmitters and receivers per node. Thus,

$$C_{net} = \alpha \sum_{l \in L} W_l + \beta \left(\sum_{n \in N} T_n + \sum_{n \in N} R_n \right) \tag{9}$$

where α is the cost of using one additional wavelength in a link and β is the cost of one transmitter or receiver.

The main constraint of the problem (equation (8)) is non linear. Thus, to be able to use linear integer programming techniques, the problem must be formulated in an alternative way that uses linear operations. Replacing B_c by equation (2) and applying the *log* function to each side of equation (8), we obtain:

$$\log\left(1 - B^{tx}(T_s)\right) + \log\left(1 - B^{rx}(R_d)\right) + \sum_{\forall l \in path(c)} \log\left(1 - B_l(W_l)\right) \geq \log\left(1 - B_{target}\right) \tag{10}$$

For a given value of traffic load, ρ, the values of the terms in equation (10) can be precomputed for all different values of W_l, T_n and R_n, and denoted by:

$$F_l^W(i) = \log\left(1 - B_l(i)\right) \qquad i = 1 \ldots p_l, \forall l \in L \tag{11}$$

$$F^{TX}(j) = \log\left(1 - B^{tx}(j)\right) \qquad j = 1 \ldots |N| - 1 \tag{12}$$

$$F^{RX}(k) = \log\left(1 - B^{rx}(k)\right) \qquad k = 1 \ldots |N| - 1 \tag{13}$$

With these values precomputed, now we are able to formulate the following integer linear programming model to compute the optimal dimensioning:

Decision variables

W_l	Number of wavelengths of link $l \in L$, $0 \leq W_l \leq p_l$.
T_n	Number of transmitters of node $n \in N$, $0 \leq T_n \leq \|N\|$-1.
R_n	Number of receivers of node $n \in N$, $0 \leq R_n \leq \|N\|$-1.

Auxiliary decision variables

$w_{l,i}$	Auxiliary binary variable, $l \in L$, $0 \leq i \leq p_l$.
$t_{n,j}$	Auxiliary binary variable, $n \in N$, $0 \leq j \leq \|N\| - 1$.
$r_{n,k}$	Auxiliary binary variable, $n \in N$, $0 \leq k \leq \|N\| - 1$.
AL_l^W	Value of $\log\left(1 - B_l(W_l)\right)$, $l \in L$.
AL_n^{TX}	Value of $\log\left(1 - B^{tx}(T_n)\right)$, $n \in N$.
AL_n^{RX}	Value of $\log\left(1 - B^{rx}(R_n)\right)$, $n \in N$.

Minimize

$$\alpha \sum_{l \in L} W_l + \beta \left(\sum_{n \in N} T_n + \sum_{n \in N} R_n \right) \tag{14}$$

Subject to

$$AL_{s_c}^{TX} + AL_{d_c}^{RX} + \sum_{\forall l \in path_c} AL_l^W \geq \log\left(1 - B_{target}\right) \qquad \forall c \in C \tag{15}$$

$$AL_l^W = \sum_{i=1}^{p_l} F_l^W(i) \cdot w_{l,i}; \qquad \forall l \in L \tag{16}$$

$$AL_n^{TX} = \sum_{j=1}^{|N|-1} F^{TX}(j) \cdot t_{n,j}; \qquad \forall n \in N \tag{17}$$

$$AL_n^{RX} = \sum_{k=1}^{|N|-1} F^{RX}(k) \cdot r_{n,k}; \qquad \forall n \in N \tag{18}$$

$$\sum_{i=1}^{p_l} w_{l,i} = 1; \qquad \forall l \in L \tag{19}$$

$$\sum_{j=1}^{|N|-1} t_{n,j} = 1; \qquad \forall n \in N \tag{20}$$

$$\sum_{k=1}^{|N|-1} r_{n,k} = 1; \qquad \forall n \in N \tag{21}$$

$$W_l = \sum_{i=1}^{p_l} i \cdot w_{l,i}; \qquad \forall l \in L \tag{22}$$

$$T_n = \sum_{j=1}^{|N|-1} j \cdot t_{n,j}; \qquad \forall n \in N \tag{23}$$

$$R_n = \sum_{k=1}^{|N|-1} k \cdot r_{n,k}; \qquad \forall n \in N \tag{24}$$

Equations (19) and (22) force that $w_{l,i} = 1$ if and only if $W_l = i$, and together with $F_l^W(i)$ from equation (11) allow us to compute $\log\left(1 - B_l(W_l)\right)$ in equation (16). Similar procedures allow us to compute the other terms of equation (10) for each connection $c \in C$.

V. Numerical results

The integer linear programming model was solved using IBM ILOG OPL Optimization Suite v6.3 in a Intel Dual-Core 2,2GHz. The maximum time required to solve each instance of the model was 3 seconds and 25 hundredths.

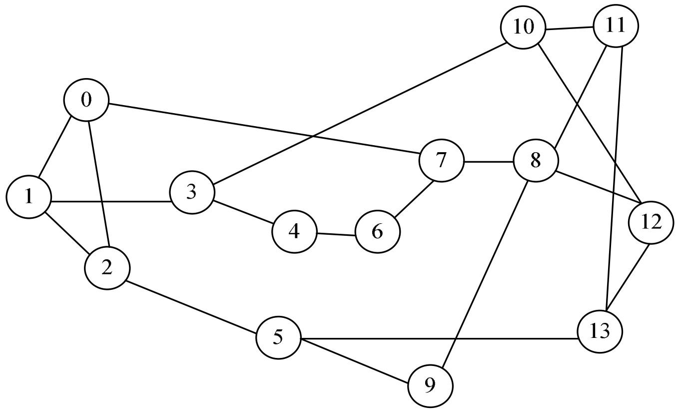

Fig. 2. NSFNet (14 nodes, 42 unidireccional links).

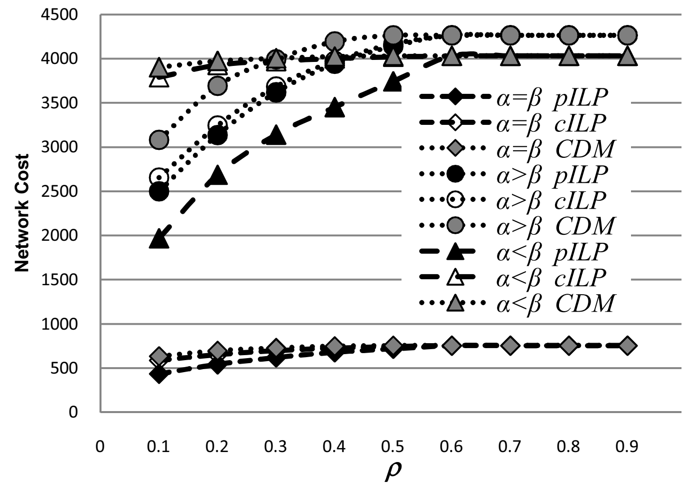

Fig. 3. Network cost as a function of ρ .

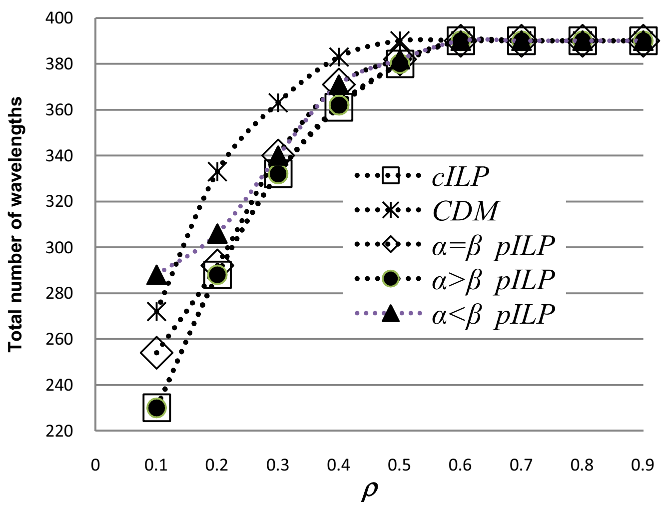

Fig. 4. Total number of wavelengths as a function of ρ.

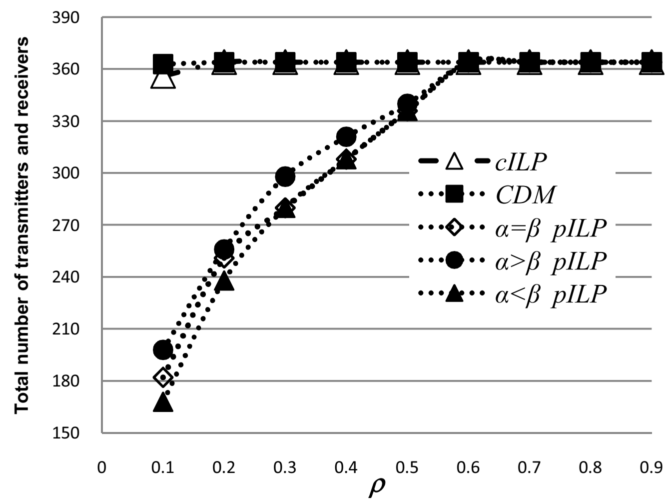

Fig. 5. Total number of transmitters and receivers as a function ρ.

All tests were run using the NSFNet topology, shown in Figure 2, and routing each connection by its shortest path.

The model was solved for three scenarios:

a) Cost of a wavelength per link (α) is equal to cost of a transmitter/receiver (β); namely, $\alpha = \beta = 1$,

b) $\alpha > \beta$; namely: $\alpha = 10, \beta = 1$,

c) $\alpha < \beta$; namely: $\alpha = 1, \beta = 10$.

Results obtained by the proposed method were compared to two dimensioning methods that do not take the number of transmitters/receivers into account. The first method is a simulation-based dimensioning algorithm proposed in [7] for dynamic WDM networks. This method is denoted as Classical Dimensioning Method, CDM, from now on. The second method consists in applying the ILP formulation presented in this paper making $\beta = 0$ in equation (9). In this way, the cost of transmitters and receivers do not impact the dimensioning of the network and we model the case of dimensioning the number of wavelengths only. This method is denoted as classical ILP (cILP) from now on. In both cases, CDM and cILP, every node n is assumed to be equipped with $\min(|N| - 1, \sum_{l \in out_n} W_l)$ transmitters and $\min(|N| - 1, \sum_{l \in in_n} W_l)$ receivers.

The target blocking probability per connection was set to 10^{-3}.

Figure 3 shows the value of the network cost (C_{net}) for the three scenarios just described ($\alpha = \beta$, $\alpha > \beta$, $\alpha < \beta$) as a function of the traffic load for the dimensioning method proposed here (pILP in the figure legend, which stands for proposed ILP) and for CDM and cILP. In the Figure, pILP is shown with a line using a black filled symbol, CDM is shown with a line using a gray-filled symbol and cILP is shown with a line using a non filled symbol.

It can be seen that for traffic loads up to 0.5, the pILP method achieves a lower network cost than CDM and cILP, that only perform link capacity dimensioning. The saving in network cost is more significant in the case of $\alpha < \beta$, where the cost of transmitters/receivers determines the network cost. However, savings can also be observed in the scenario $\alpha > \beta$, where the cost of wavelengths is much more significant than the cost of transmitters/receivers. For traffic loads higher than 0.5, the three methods achieve the same network cost as for such high loads it is necessary to assign the maximum number of transmitters and receivers ($|N| - 1$) to all network nodes to meet the target blocking probability.

To study the impact of the proposed dimensioning method in the number of resources required in nodes and links separately, Figures 4 and 5 show the total number of wavelengths ($\sum_{l=0}^{|L|-1} W_l$) and the total number of transmitters/receivers ($\sum_{n=0}^{|N|-1} T_n + \sum_{n=0}^{|N|-1} R_n$) required by the three methods as a function of the traffic load for the three scenarios described.

From Figure 3 it can be seen that the cILP method obtains a lower requirement of wavelength than the CDM for traffic loads under 0.6, as it uses a technique that guarantees an optimal solution. It can also be seen that the number of wavelengths required by the pILP is extremely close to that of the cILP method for all scenarios, except when $\alpha < \beta$. In this scenario the cILP decreases the network cost at the expense of increasing the number of cheaper components (wavelengths).

From Figure 4 it can be seen that for low values of traffic load, the total number of transmitters and receivers required by the CDM and cILP mehods is up to the double of the number required by the pILP method. For traffic loads higher tna 0.6, the number of transmitters/receivers of the three methods becomes equal to the maximum ($|N| - 1$) to satisfy the blocking probability requirement.

VI. Summary

In this paper an integer linear programming formulation for dimensioning of dynamic WDM networks was proposed. Unlike previous dimensioning techniques, that consider the link dimensioning or the transmitters/receivers dimensioning separately, the proposed method takes into account both the number of wavelengths per link and the number of transmitters and receivers of network nodes.

Three component cost scenarios were studied: wavelength cost higher than/equal to/lower than transmitter/receiver cost. Results show that the method proposed in this paper achieved a lower network cost than the classical method of just dimensioning the link capacity of a network for all the scenarios studied; with a highest saving in the case when the cost of transmitters/receivers is higher than that of the wavelengths. In this case, up to 50% of cost saving regarding the classic approach of dimensioning just the network links was obtained. Compared to a heuristic method proposed in [7] for dimensioning the link capacity of dynamic WDM networks, the method proposed not only decreased the number of transmitters/receivers required at each network node but also the required link capacity.

These results show that network operators/designers should take into account the number of transmitters/receivers when dimensioning a dynamic WDM network, as they can obtain significantly lower network cost than just allocating the maximum number of them to each node.

VII. Acknowledgments

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