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Robust Planning for an Open-Pit Mining Problem under Ore-Grade Uncertainty

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Abstract

Open-pit mining production planning is a risky problem: operation costs are considerable, many parameters are inherently subject to uncertainty and, moreover, the mining operation can only be done once. In this work we address uncertainty in the ore-grade, where we only assume the availability of an i.i.d. sample of the joint distribution of ore-grade in the blocks of the mining site. We consider an open-pit mining problem involving extraction and processing decisions under capacity constraints. We apply and compare the risk-hedging performance of three approaches for optimization under uncertainty: Value-at-Risk, Conditional Value-at-Risk and a proposed robust optimization approach. The latter is shown to have desirable risk-averse properties. Computational results on one small size vein-type mine are shown.

Keywords: Open-pit mining, Robust Optimization, Risk-averse optimization

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1 Introduction

Uncertainty plays a major role in open-pit mining production planning. A crucial parameter subject to uncertainty is the ore-grade of each block in the ore field: high costs of prospective drillings makes necessary to limit them and rely on interpolations with geostatistical techniques. Moreover, production planning problems are essentially *one-shot* problems: the exploitation of the ore body is done only once. For all this, a pure maximization of expected profits is questionable. Previous works on this subject have considered simulations, maximization of expected value and chance constrained optimization (see [4]).

In this work we study the effect of ore-grade uncertainty on the final profit of the mining operation. We assume that the ore-grades can be accessed by i.i.d. samples, in this way we make independent the risk analysis from the geostatistical technique that models the joint ore-grade distribution. We consider a model of open-pit planning under capacity constraints in which, on a single-period time horizon, the decision consists on which blocks of the mine are going to be extracted, and among the extracted, which ones are going to be processed. These simplifications are for the sake of simplicity and understanding, but the approach is readily extendible to multi-process and multi-period.

The rest of this text is organized as follows. §2 presents the extraction and processing planning problem we consider. §3 presents three common models for risk/robust optimization.§4 presents some performance comparisons for these models in a vein-type mine. Finally, §5 presents the main conclusions and remarks of our work.

2 Problem description

We consider a single-period horizon production planning problem, and for the time being we will assume that the value of all parameters is known and is not subject to uncertainty. We use the traditional block model for open-pit mining, and consider the following IP formulation. Define $\mathcal{B}$ as the set of blocks in the ore-field and $\mathcal{P} \subseteq \mathcal{B} \times \mathcal{B}$ the set of *precedence constraints* for extraction; that is, $(a, b) \in \mathcal{P}$ iff to extract b one must extract a. Variables are $x^p, x^e \in \{0,1\}^{\mathcal{B}}$, where $x^e_b = 1$ iff block b is extracted, and $x^p_b = 1$ iff block b is processed. Only extracted blocks can be processed. Each block has a given tonnage, and there is a maximum extraction tonnage and a maximum processing tonnage. The profit function is $\Pi_\rho(x^e, x^p) = (w \cdot \rho)' x^p - (c^p)' x^p - (c^e)' x^e$, where $c^e, c^p \in \mathbb{R}^{\mathcal{B}}_+$

are the extraction and processing costs, $\rho \in \mathbb{R}_+^{\mathcal{B}}$ is the ore grade, and $w > 0$ the unitary value of the processed ore. Defining X as the set of feasible production plans, the problem is: max $\Pi_\rho(x^e, x^p)$ s.t. $(x^e, x^p) \in X$. Note that $\Pi_\rho(x^e, x^p)$ is linear in (x^e, x^p) for all ρ, and that X does not depend on ρ.

3 Assessing uncertainty

From this point onward, we will assume that the vector $\widetilde{\rho} \in \mathbb{R}_+^{\mathcal{B}}$ of ore-grades is a joint random vector following some (possibly unknown) stochastic distribution, but that we can obtain i.i.d. samples of it. Note however that this uncertainty only appears in the objective function.

3.1 Minimization of Value-at-Risk

Our first model is minimization of Value-at-Risk (VaR) of the profit. For a general random variable L representing a loss (then $-L$ is a profit), its VaR with risk level $\epsilon \in [0, 1)$ is defined as $\mathrm{VaR}_\epsilon(L) := \min\{\zeta \ : \ \mathbb{P}(L \leq \zeta) \geq 1 - \epsilon\}$. Hence, $t = \mathrm{VaR}_\epsilon(L)$ implies that, with a confidence level of $(1 - \epsilon)$, losses will not exceed the threshold t; therefore lower values of t are more desirable.

For every feasible production plan $(x^e, x^p) \in X$ the profit of the plan, $\Pi_{\widetilde{\rho}}(x^e, x^p)$, is a real random variable. Given a desired risk level $\epsilon \in [0, 1)$, the minimization of VaR model consists of

$$\min\{\ \zeta \ : \ \mathbb{P}(-\Pi_{\widetilde{\rho}}(x^e, x^p) \leq \zeta) \geq 1 - \epsilon, \quad (x^e, x^p) \in X\ \} \tag{1}$$

Computability issues and non-convexity of the chance constraint makes this model, as stated, untractable. However, using an i.i.d. sample $\{\rho^i\}_{i=1}^N$ of $\widetilde{\rho}$ we can approximate $\mathbb{P}$ with the in-sample frequentist probability, obtaining a common MILP formulation [8]. This approach, also known as *Sample Average Approximation* (SAA) of a chance constraint, is rather popular and has been widely studied (see e.g. [6]). Remarkably, under mild conditions, as the sample size grows, the approximated problem's objective value and feasible set converges to its respective counterparts of the real problem [6].

3.2 Minimization of Conditional Value-at-Risk

The second used model is minimization of Conditional Value-at-Risk (CVaR) of the profit. Given a desired risk level $\epsilon \in (0, 1]$, if the atom-less random variable L represents a loss (then $-L$ a profit), its CVaR with risk level ϵ is defined as $\mathrm{CVaR}_\epsilon(L) := \mathbb{E}\left[L | L \geq \mathrm{VaR}_\epsilon(L)\right]$ (see [7] for details). The minimization of

CVaR_ϵ of the profits consists on

$$\min\{\ \zeta + \frac{1}{\epsilon}\mathbb{E}\left[(-\Pi_{\widetilde{\rho}}(x^e, x^p) - \zeta)^+\right]\ :\ (x^e, x^p) \in X,\ \zeta \in \mathbb{R}\ \} \tag{2}$$

Unlike VaR, CVaR is a coherent risk measure in the sense of [1]. Minimization of CVaR is therefore a risk-averse optimization approach. Note also that the model is equivalent to maximize expected profits when $\epsilon = 1$.

As before, we use an i.i.d. sample of $\widetilde{\rho}$ to approximate $\mathbb{E}$ by the in-sample average, i.e. we use the SAA of the expected value. SAA approximation of expected value has also been widely studied (see [5]) and under mild conditions there is convergence of the approximated problem to the real one (see [8]).

3.3 A Robust optimization approach

We propose a robust optimization model based on the availability of an i.i.d. sample $\{\rho^i\}_{i=1}^N \in \mathbb{R}_+^{\mathcal{B}}$ of the joint ore-grades. For a desired risk-level $\epsilon \in [0, 1]$ we consider the set $\mathcal{U}_\epsilon := \bar{\rho} + (1 - \epsilon)\ \big(\mathrm{conv}(\{\rho^1, \ldots, \rho^N\}) - \bar{\rho}\big)$, where $\bar{\rho} := \sum_{i=1}^N \rho^i/N$. Intuitively, as ϵ grows from 0 to 1, the convex hull of the sample colapses to the mean ore-grade. The proposed approach, which we denote *Modulated Convex-Hull model* (MCH), consists in

$$\max_{(x^e,\ x^p)\in X}\ \min_{\rho\ \in\ \mathcal{U}_\epsilon}\ \Pi_\rho(x^e, x^p) \tag{3}$$

An equivalent MILP formulation is obtained using the vertices of $\mathcal{U}_\epsilon$ and affine-linearity of Π_ρ in ρ. Note that we are moving between maximizing worst in-sample profit (if $\epsilon = 0$) and maximizing the in-sample mean profit (if $\epsilon = 1$). This enables the decision maker to choose the desired balance between conservativeness and the traditional stochastic approach of maximizing expected profits. Finally, we present a close relationship between the proposed model, minimization of CVaR and risk-averse optimization.

Proposition 3.1 (i) *Let $\epsilon \in [0, 1]$, and let $(\Omega, \mathcal{F}, \mathbb{P}_N)$ be a finite, equiprobable probability space of cardinal N. For all loss function $Z \in \mathcal{L}^\infty(\Omega, \mathcal{F}, \mathbb{P}_N)$ define*

$$\mu_\epsilon(Z) \quad := \quad \epsilon\ CVaR_1(Z) + (1 - \epsilon)\ CVaR_{1/N}(Z)$$

Then μ_ϵ is a coherent risk measure. Moreover, it is law-invariant and comonotonic, in the sense of [2].

(ii) *Given an i.i.d. sample $\rho^1, \ldots, \rho^N$ defining an equiprobable space, the proposed approach (3) is equivalent to $\min_{(x^e,\ x^p)\in X} \mu_\epsilon\left(\Pi_\rho(x^e, x^p)\right)$*

4 Computational results

We apply the three approaches on a small size (approx. 16000 blocks of $10\,m \times 10\,m \times 10\,m$) vein type mine, for which an i.i.d. sample of 1000 joint ore-grades obtained using [3]. We take a random sample of $N < 1000$ ore-grades and solve the three models for several risk levels ϵ, obtaining thus several plans. For each plan, we obtain its *in-sample profits* via evaluating its profit on each of the N in-sample ore-grades; analogously, *full-sample profits* are obtained when evaluating the profit of the plan on each of the 1000 ore-grades available. Histograms of in- and full-sample profits are shown in Figures 1 and 2. For the sake of comparability, all figures have the same bound on x- and y-axis, and mean and std. deviation are shown over each curve.

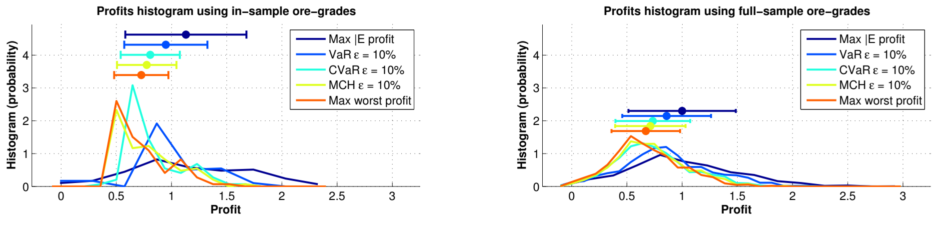

Fig. 1. Profits histograms obtained using a size $N = 100$ sample

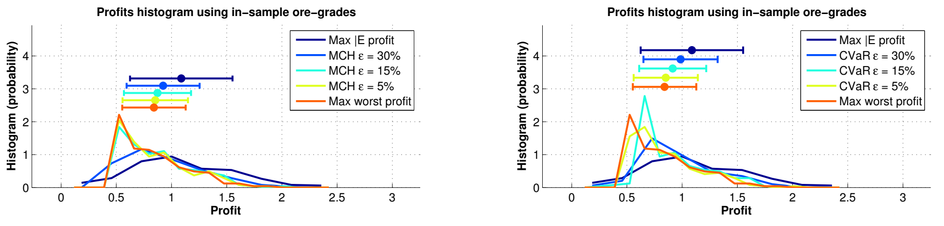

Fig. 2. Profits histograms obtained using a size $N = 180$ sample

5 Conclusions and remarks

- VaR model shows riskier results than maximizing expected in-sample profits, since the former allocates higher probability in the worst profits than the latter does. In consequence, minimizing VaR does not show appropriate under a risk-averse perspective.
- CVaR and the proposed MCH model have attractive theoretical properties under i.i.d. sampling; namely, coherence, law-invariance and comonotonicity (see [2]). In practice, CVaR and MCH models show a conservative performance, since worst profits are attained with lower probability

than when maximizing mean in-sample profits (see Figure 2). However, this is done at the expense of having lower probability of attaining high profits.

- Theoretically, modeling uncertainty using i.i.d. sampling shows sound: for VaR and CVaR models convergence as the sample size grows is assured under mild conditions. In practice, however, the in-sample behaviour might be lost once evaluated in the *true* distribution, as shown in Figure 1.
- Future work includes statistical validation, for each model, of the in-sample solution as a solution of their respective *r*eal stochastic models, for example using [8, Chap. 5]. Another area of future work is setting the processing decision as a *recourse*, or *second-stage*, variable.

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