# A two-stage stochastic model for open pit mine planning under geological uncertainty

Eduardo Moreno[1]*, Xavier Emery[2,3], Marcos Goycoolea[4], Nelson Morales[2,3], Gonzalo Nelis[2,3]
[1]*Faculty of Engineering and Sciences, Universidad Adolfo Ibáñez, Chile;* [2]*Department of Mining Engineering, Universidad de Chile, Chile;* [3]*Advanced Mining Technology Center, Universidad de Chile, Chile;* [4]*School of Business, Universidad Adolfo Ibáñez, Chile.*
*Corresponding author: eduardo.moreno@uai.cl

We propose a two-stage production scheduling model for open pit mine planning under geological uncertainty that considers different conditional simulations of a mineral deposit, based on the information from drill holes. In a first stage, the scheduling decision is taken, assigning an extraction period of each region (block, bench-phase or similar) of the deposit. In a second stage, when the true ore grade is revealed, the model decides how to treat each individual block in that region. Our proposed integer-programming model can be reformulated as a large-scale precedence constrained knapsack problem that can be (near-optimally) solved using decomposition techniques. This approach allows us to solve real instances of the problem in a few hours.
We apply this model to a copper deposit in Chile, using different number of drill holes to generate scenarios. We compare the resulting NPV from the deterministic solution, the best-possible plan for each scenario, and our proposed model. Computational experiments show that, in these data, the proposed two-stage stochastic model produces more robust mine plans, with an improvement on the obtained NPV of 4% to 7% for scenarios with high geological uncertainty.

## Introduction

The central concern of strategic mine planning is to construct a tentative life-of-mine production schedule specifying which part of a mineral deposit should be extracted, and when and how it should be extracted to maximize value and satisfy operational constraints. In early stages of a mining endeavor, such a plan serves to anticipate the cash flows of a project (capacity investments and production goals) and, thus, is critical to investors. Because early decisions are binding and, hence, critical to long-term profits, it is widely acknowledged that, of all stages in the life of a mining project, strategic planning most significantly impacts final profits.

Strategic mine planning is a complex optimization problem made daunting by geological uncertainty (distribution of mineral resources over space), and the scale of practical problems (amount of data and number of decisions involved). The tasks of creating a production schedule and modeling the inherent uncertainty are extremely challenging on their own.

Mine planning methodologies developed to date by industry and academia fail to address both the concern of scale and uncertainty. Current methods assume that the values of geological variables are known, and risk management is limited to analyzing different settings of these values (scenarios) through trial and error. Moreover, current models do not consider that agents change their decisions over time as uncertainty unfolds, resulting in conservative solutions.

Multistage stochastic models are ideal for settings in which decisions can be updated over time as information is progressively revealed. The most common approach to date typically assumes that strategic decisions made at the beginning of a planning period are binding and cannot be changed as scenarios are revealed (i.e. no recourse; see [1], and [2]). This can result in very conservative solutions (see [3] and [4] for discussions). Current methods typically use expected values of net present values (NPV), assuming decision makers are risk neutral [5]. Other methods use heuristics to find feasible solutions of simple stochastic models [6][7]. Multistage stochastic models for mine planning have been proposed by [8] and [9]. While these works incorporate some desirable features - such as modeling endogenous geological uncertainty in the former, and risk metrics for price uncertainty in the latter - they do not address the issue of scalability (e.g., via decomposition), and as a result they can only deal with relatively small problems.

In this paper, we propose a two-stage stochastic model for the problem. In a first stage, the scheduling decision is taken, assigning an extraction period of each region (block, bench-phase or similar) of the mine. In a second stage, when the true ore grade is revealed, the model decides how to treat each individual block in that region. An important feature of this model is that it is scalable, and can be solved to near-optimality even for large-scale instances with several scenarios.

### Optimization models for mine planning

In the last decade, several works have shown that it is possible to formulate and solve a (deterministic) production-scheduling problem using mixed-integer programing models (MIP). In this section we discuss a generic deterministic model, which allows us to introduce the notation utilized in this work.

Usually, two types of variables are used in MIP models: variables $x_{ct}$ indicating that a "cluster" of blocks $c$ is extracted at time $t$. A cluster of blocks can be a single block, a mining-block, a bench-phase, or any other aggregation of blocks that should be extracted simultaneously. The second type of variable $y_{b,d,t}$ indicates the fraction of each block $b$ that is sent to destination $d$ at time $t$. Using these two types of variables, we can formulate a generic MIP model for the problem as following:

$$\max \sum_{t \in T} \delta^t \left( \sum_{b \in B} \sum_{d \in D} p_{b,d} y_{b,t,d} - \sum_{c \in C} c_c x_{c,t} \right)$$

$$x_{c,t} = \sum_{d \in D} y_{b,d,t} \quad \forall b \in B_c, \forall c \in \mathcal{C}, t \in T \tag{1}$$

$$\sum_{c \in C} w_c^r x_{c,t} \leq C_t^r \quad \forall t \in T, r \in R \tag{2}$$

$$\sum_{b \in B} \sum_{d \in D} w_{b,d}^{r'} y_{b,t,d} \leq C_t^{r'} \quad \forall t \in T, r' \in R' \tag{3}$$

$$Ax + By = f \tag{4}$$

$$x \in \Omega, y \geq 0 \tag{5}$$

The objective function consider the profit $p_{b,d}$ of each block sent to destination $d$, and the cost $c_c$ of extracting cluster $c$. Additionally, a discount factor $\delta^t$ is applied to the objective to compute the net present value (NPV) of the project.

Constraints (1) link the extraction and processing decision, indicating that all blocks in each extracted cluster should be sent to a destination in the same proportion. Constraints (2) and (3) represent capacity constraints of a recourse $r$ over the extraction and processing decision, respectively. Additional arbitrary constraints can be added in (4). Finally, the constraint $x \in \Omega$ in (5) imposes operational constraints over the extraction decisions of clusters. For example, if cluster are individual blocks, then $\Omega$ should include pit-slope precedence constraints. If cluster are bench-phases, then $\Omega$ should include precedence constraints between different phases and benches. $\Omega$ also includes the integrality constraints over extraction variables.

### A two-stage optimization model for geological uncertainty

To consider the geological uncertainty of the block model in the mine-planning problem, we propose what is known as a *two-stage stochastic model*. In a two-stage stochastic model, a set of decisions is taken (first stage) without knowing the real value of the random parameters. After executing this first stage, the real value of these parameters is revealed, and recourse decisions (second stage variables) are taken. The model should consider all possible second stage scenarios in order to decide the optimal first stage decisions.

In the case of open-pit production scheduling, the first stage decisions correspond to the extraction decisions ($x_{ct}$ variables), which should be taken considering the geological uncertainty of the deposit. After extracting a set of blocks, the true grades and other parameters of the extracted blocks are revealed, so the processing decisions ($y_{b,d,t}$

variables) correspond to the second stage decisions. Hence, for the generic problem presented in the previous section, we assume that the uncertainty affects the profit $p_{b,d}(\xi)$ or the resources $w^{r'}{}_{b,d}(\xi)$ of each block. Hence, our two-stage stochastic model is given by the following problem:

$$
\begin{aligned}
\max \sum_{t\in T}\delta^t\left(-\sum_{c\in C} c_c x_{c,t}\right) + \mathbb{E}_\xi(Q(x,\xi)) \\
\sum_{c\in C} w_c^r x_{c,t} \leq C_t^r \quad \forall t\in T, r\in R \\
x\in\Omega
\end{aligned}
$$

which includes the extraction decisions and its resources and operational constraints. The objective function includes the expected value of the second stage, which is given by the processing decisions and its corresponding constraints. This second stage problem, which depends on the first stage decisions *x* and a random component *ξ*, is given by the subproblem:

$$
\begin{aligned}
Q(x,\xi) := \max \sum_{t\in T}\delta^t \sum_{b\in B}\left(\sum_{d\in D} p_{b,d}(\xi) y_{b,t,d}\right) \\
x_{c,t} = \sum_{d\in D} y_{b,d,t} \quad \forall b\in B_c, \forall c\in\mathcal{C}, t\in T \\
\sum_{b\in B}\sum_{d\in D} w_{b,d}^{r'}(\xi) y_{b,t,d} \leq C_t^{r'} \quad \forall t\in T, r'\in R' \\
Ax + By = f \\
y \geq 0
\end{aligned}
$$

Since in practice, there are not known probabilistic distribution for the grades and resources, we need to rely in an approximation of the expected value, by using simulations of the mineral deposit. This technique, known as Sample Average Approximation [10], allows approximating the expected value of the second stage $E(Q(x,\xi))$ by an average of the value of these subproblems $Q(x,\xi_s)$ over a set of sampled scenarios S. Note that in this case, the processing decisions ($y_{b,d,t}$ variables) will depend on the scenario that is considered. Hence, we can add an additional subindex to the *y* variables and rewrite the complete problem into the following model:

$$
\begin{aligned}
\max \sum_{t\in T}\delta^t\left(\frac{1}{|S|}\sum_{s\in S}\sum_{b\in B}\sum_{d\in D} p_{b,d,s}\, y_{b,t,d,s} - \sum_{c\in C} c_c x_{c,t}\right) \\
x_{c,t} = \sum_{d\in D} y_{b,d,t,s} \quad \forall b\in B_c, \forall c\in\mathcal{C}, t\in T, s\in S \\
\sum_{c\in\mathcal{C}} w_c^r x_{c,t} \leq C_t^r \quad \forall t\in T, r\in R \\
\sum_{b\in B}\sum_{d\in D} w_{b,d,s}^{r'} y_{b,t,d,s} \leq C_t^{r'} \quad \forall t\in T, r'\in R', s\in S \\
Ax + By_s = f \quad \forall s\in S \\
x\in\Omega, y\geq 0
\end{aligned}
$$

where $p_{b,d,s}$ and $w_{b,d,s}$ correspond to the value of these random parameters in the scenario *s*.

This model multiplies the number of $y$ variables and constraints (1), (3) and (4) of the deterministic model by the number of scenarios. However, it is still possible to reformulate this problem to convert it into a General Precedence Constrained Problem, suitable to be solved using the BZ algorithm [11].

### Computational Experiment

We evaluate the potential of using a two-stage stochastic model over a real copper deposit located in northern Chile. We use the information from drill holes to generate two sets of 50 scenarios by conditional simulations using the turning bands algorithm [12]. A first set was generated using 76 drill holes, obtaining scenarios with high uncertainty. The second set has been generated using 753 drill holes, so the resulting scenarios have a lower uncertainty. The economic parameters of the mine are presented in Table 1. The constraints of the problem are a maximum mining and processing capacity for every period.

| | | | | | |
|---|---|---|---|---|---|
| Copper price | 2.1 US$/lb | Recovery | 85% | # blocks in ult. pit | 145,858 |
| Mine cost | 2.5 US$/ton | Mining capacity | 43.8 Mton/year | # phases | 4 |
| Cost Flotation | 10 US$/ton | Plant capacity | 23.725 Mton/year | # benches | 30 |
| Cost R&F | 0.25 US$/lb | Discount factor | 10% | # bench-phases | 117 |

Table 1: Parameters of the mine instance utilized

We solve two type of problems. The first problem (named "block-level scheduling") schedules individual blocks along the time, subject to slope constraints. Hence, cluster set $C$ corresponds to individual blocks and the set $\Omega$ includes slope-precedence constraints and binary constraints for the variables $x$. The second problem (named "Bench-phase scheduling") uses a predefined set of four phases computed using Whittle's Milawa Balanced algorithm, so the set of clusters $C$ corresponds to the bench-phases of the mines, and the set $\Omega$ includes the usual constraints between bench-phases.

For each problem, and for each set of scenarios, we benchmark the results of three different approaches. The first approach is the stochastic model presented in the previous section, considering the 50 available scenarios. Recall that this model returns a unique extraction sequence given by $x$ variables, and a different policy for processing blocks on each scenario (given by $y$ variables). The second approach is the usual deterministic approach, where a unique block model is obtained by averaging 50 scenarios, and solved using the deterministic MIP formulation. However, in order to do a fair comparison with the stochastic model, we evaluate the extraction sequence obtained by the deterministic model, allowing to change the processing decisions in each scenario. Finally, our third approach, named "Cristal Ball", consists in solving the deterministic model for each scenario individually. Note that this approach produces a different extraction sequence for each one of the 50 scenarios, so it cannot be used in practice. However, it provides an upper bound on the best possible NPV that can be obtained in each scenario. For each approach, and in order to avoid numerical misinterpretations, we present the optimal NPV value of its linear relaxation, which is obtained using a modified version of the Bienstock-Zuckerberg algorithm [11]. However, for each case an integer solution with an integrality gap smaller than 1% can be obtained.

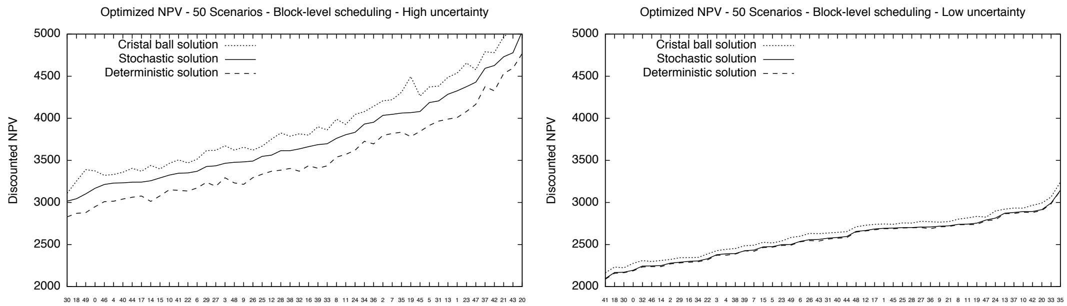


Figure 1: NPV obtained by each approach at block-level scheduling. Left: case when 76 drill holes are available. Right: case when 753 drill holes are available.

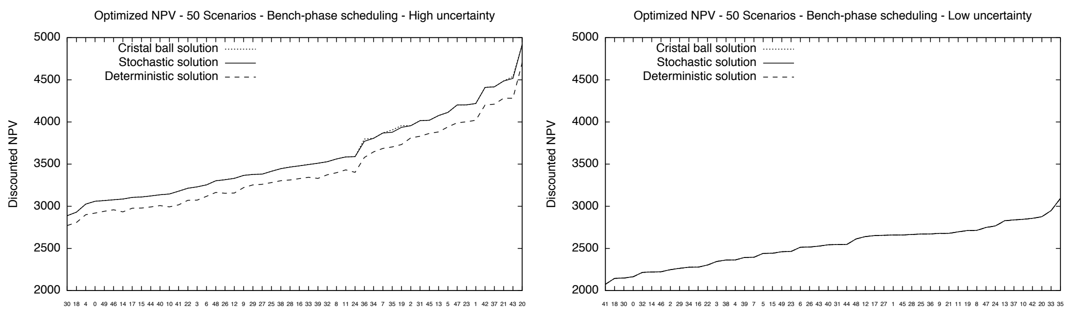


Figure 2: NPV obtained by each approach at bench-phase scheduling. Left: case when 76 drill holes are available. Right: case when 753 drill holes are available

Figure 1 and 2 show the NPV obtained by each approach on the scenarios with high uncertainty (left) and low uncertainty (right) when we schedule blocks and bench-phases, respectively.

Note that the resulting NPV of the stochastic and the deterministic models with scenarios of low uncertainty (right side of Figures 1 and 2) are very similar, obtaining almost the same profit on each scenario. This is an expected behavior, that shows that with low uncertainty the deterministic model is near-optimal, so the stochastic model cannot do anything better. Interestingly, the stochastic model is not worse than the deterministic model for any scenario, which is an unexpected result that shows that the use of the stochastic model even for low uncertainty scenarios is still recommended.

On scenarios with high uncertainty, the stochastic model shows a higher benefit. In average, the profit obtained by the stochastic model is 6.47% and 4.69% higher than the determinist model for blocks-level and bench-phase scheduling, respectively. Moreover, the stochastic model captures most of the uncertainty of the problem. In fact, for the block-level scheduling, the stochastic model captures between a 40% to 68% of the difference between the NPV of the deterministic model and the best-possible NPV for each scenario. Results of the stochastic model are even better for the bench-phase scheduling, where the stochastic model is very similar to the best possible solution on each scenario. In fact, the difference between the NPV of the stochastic model and the best possible solution is less than 0.71%, with an average of 0.06%.

In Figure 3 we compare the tonnage movement and the average profit per ton at the plant for the stochastic and deterministic solutions for bench-phase scheduling, in the scenarios 20 and 30 of high uncertainty, where the best and worst NPV for this problem are obtained. Note that the stochastic solution uses the complete mining capacity, extracting the mine in 9 periods. In contrast, the deterministic solution does not use the complete mining capacity, requiring one more period. The deterministic solution has this extraction sequence because in the average scenario it could use the 100% of the plant capacity. However, we can see that even for the best scenario, the deterministic solution does not have sufficient ore to use the full plant capacity. In fact, deterministic solution utilizes 85.8% and

76.2% of the plant capacity for the best and worst scenario, respectively. On the other hand, the stochastic solution extracts more material per period, having more available ore to be sent to the plant, which gives more flexibility. This can be seen in the use of the plant capacity, which is a 92.0% and 84.5% for the best and worst scenario, respectively. For the same reason, the average grade of the material sent to the plant is higher for the stochastic solution, obtaining an NPV 4.6% and 4.2% better than the deterministic solution for the best and worst scenario, respectively.

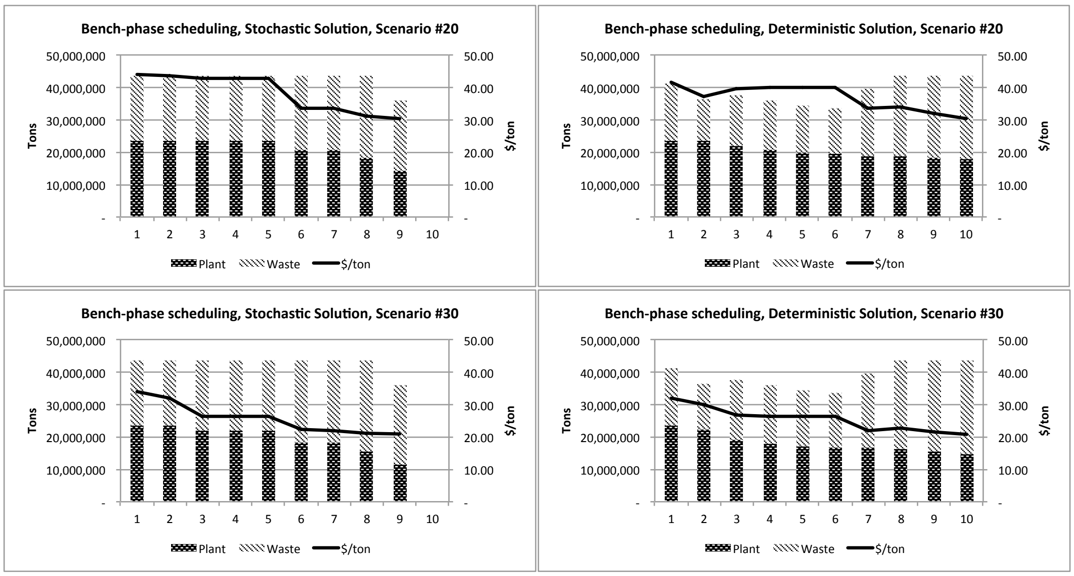


Figure 3: Bench-phase scheduling for the best (#30) and the worst (#20) scenario of high uncertainty.

Finally, in order to provide a more realistic use of this model in the industry, we compare the schedule obtained by the deterministic and stochastic models for scenarios with high uncertainty, but we evaluate them over the scenarios with low uncertainty. The resulting NPV are presented in Figure 4, for both block-level (left) and bench-phase (right) cases. We also include the Cristal ball solution for low uncertainty scenarios, to show the best possible NPV that can be obtained in each case. Figure 4 shows that the stochastic solution is very close to the best possible solution in both cases. For the block-level scheduling, the stochastic solution has an expected NPV 6.47% higher than the deterministic solution, and 4.62% lower than the average NPV of the best possible solution for each scenario. For the bench-phase scheduling, the stochastic solution and the best possible solutions have basically the same average NPV, which is 3.8% better than the deterministic solution.

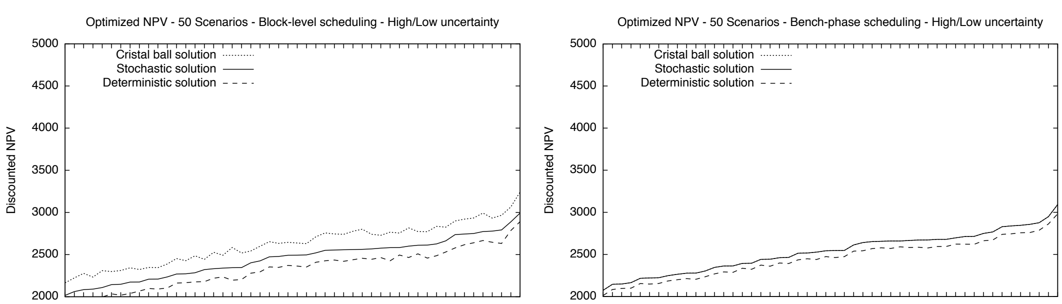


Figure 4: NPV of the solution for block-level (left) and bench-phases (right) scheduling for the case when 76 drill holes are available, but evaluated in the scenarios when 753 drill holes are available.

## Conclusions

We presented a stochastic optimization model for mine planning problems that consider the geological uncertainty of the mineral deposit. The proposed model provides an extraction sequence for all periods, and the processing decisions are taken knowing the true grade of the extracted material. The model is also scalable, being able to solve an instance with hundreds of thousands blocks and 50 scenarios using state-of-art decomposition methods. We also benchmarked the stochastic solutions with the classical deterministic approach and the best-possible solution for each scenario. The stochastic solution shows a considerable improvement in the NPV over the deterministic solution, and very close to the best possible NPV that can be obtained.

The proposed stochastic model is sufficiently general, and can be applied to more complex problems with different constraints and objectives. For example, further studies can be done introducing a stockpile (like in [13]) to the stochastic model or incorporating production targets

## Acknowledgments

Authors acknowledge the financial support of CONICYT PIA Anillo ACT1407.

## References

[1] S. Ramazan, R. Dimitrakopoulos, “Traditional and new MIP models for production scheduling with in-situ grade variability”, International Journal of Surface Mining, Reclamation and Environment, 2004, 18(2):85–98.

[2] J. Gholamnejad, M. Osanloo, “Using chance constrained binary integer programming in optimizing long term production scheduling for open pit mine design”, Mining Technology: IMM Transactions section A, 2007, 116(2):58–66.

[3] D. Espinoza, M. Goycoolea, E. Moreno, G. Muñoz, M. Queyranne, “Open pit mine scheduling under uncertainty: a robust approach”, Proceedings of APCOM 2013, 2013:492-501

[4] S. A. Abdel Sabour, R. Dimitrakopoulos, “Incorporating geological and market uncertainties and operational flexibility into open pit mine planning”, Journal of Mining Science, 2011, 47(2):191–201.

[5] A. Newman, E. Rubio, R. Caro, A. Weintraub, “A review of operations research in mine planning”, Interfaces, 2010, 40(3):222–245.

[6] A. Lamghari, R. Dimitrakopoulos, J.A. Ferland, “A variable neighbourhood descent algorithm for the open-pit mine production scheduling problem with metal uncertainty”, Journal of the Operational Research Society, 2013, 65(9):1305–1314.

[7] A. Lamghari, R. Dimitrakopoulos, “A diversified Tabu search approach for the open-pit mine production scheduling problem with metal uncertainty”, European Journal of Operational Research, 2012, 222(3):642–652.

[8] N. Boland, I. Dumitrescu, G. Froyland, “A multistage stochastic programming approach to open pit mine production scheduling with uncertain geology”, 2008, Optimization Online (http://www.optimization-online.org*)*.

[9] A. Alonso-Ayuso, F. Carvallo, L. Escudero, M. Guignard, J. Pi, R. Puranmalka, A. Weintraub, “Medium range optimization of copper extraction planning under uncertainty in future copper prices”, European Journal of Operational Research, 2014, 233(3):711–726.

[10] A. J. Kleywegt, A. Shapiro, T. Homem-de-Mello, "The sample average approximation method for stochastic discrete optimization", SIAM Journal on Optimization, 2002, 12(2):479-502.

[11] G. Muñoz, D. Espinoza, M. Goycoolea, E. Moreno, M. Queyranne, O. Rivera, “A study of the Bienstock-Zuckerberg algorithm: applications in mining and resource constrained project scheduling”, arXiv:1607.01104, 2016.

[12] X. Emery, C. Lantuéjoul, “TBSIM: A computer program for conditional simulation of three-dimensional Gaussian random fields via the turning bands method”, Computers & Geosciences, 2006, 32(10):1615-1628.

[13] E. Moreno, M. Rezakhah, A. Newman, F. Ferreira, “Linear Models for Stockpiling in Open-pit Mine Production Scheduling Problems”, European Journal of Operational Research, 2016, 260(1):212-221.