Risk control in ultimate pits using conditional simulations

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Abstract

In this work we study how to incorporate risk control to the generation of ultimate pits when orebodies are modeled through a finite number of conditional simulations. To control risk we consider a chance constraint on the value of the ultimate pit. We incorporate this measure into the generation of ultimate pits by solving a stochastic programming version of the ultimate pit problem. We compare this stochastic programming problem to previous approaches such as generating the ultimate pit for each simulation and the hybrid pit approach introduced by Whittle and Bozorgebrahimi. We also study the effect of using different number of simulations in the generation and evaluation of ultimate pits.

Biography

Juan Pablo Vielma is a Ph.D. candidate at the H. Milton Stewart School of Industrial & Systems Engineering at the Georgia Institute of Technology. His doctoral dissertation studies mixed integer programming approaches for nonlinear and stochastic programming. After graduating in the summer of 2009 he will spend one year at the IBM Thomas J. Watson Research Center as the 2009 Herman Goldstine Postdoctoral Fellow. He then expects to join the department of Industrial Engineering of University of Pittsburgh in 2010 as an assistant professor. In 2007 he received the first place in the INFORMS Optimization Society Student Paper Prize and he is currently the Vice-Chair (Chair Elect) of the INFORMS section on Energy, Natural Resources, and the Environment.

Introduction

The consideration of geological uncertainty in open pit mining has received significant attention in the last decade. A now standard tool to model this kind of uncertainty is the use of conditional simulations of orebodies (e.g. Dowd, 1997; Dimitrakopoulos, 1998; Benndorf and Dimitrakopoulos, 2007) and its incorporation into the design and evaluation process of mines has been the topic of many recent papers. Dimitrakopoulos et al. (2002) uses conditional simulations to show that designs obtained from a single orebody model, such as ordinary kriging, can have a high risk of significantly deviating from their financial projections. Dimitrakopoulos et al. (2007) incorporate conditional simulations into the design process by evaluating several plans under different orebody models and selecting the one that presents the best performance indicators. Menabde et al. (2007) incorporate conditional simulations into a stochastic integer programming problem that maximizes the expected NPV of an extraction sequence and Ramazan and Dimitrakopoulos (2007) develop a similar model that also includes penalties for not meeting production targets. Whittle and Bozorgebrahimi (2007) introduces hybrid pits as a way to incorporate conditional simulations to the generation of ultimate of final pits and Golamnejad et al. (2006) incorporates probabilistic constraints to reduce risk in long term production scheduling. Other related publications include Ramazan and Dimitrakopoulos (2004), Godoy and Dimitrakopoulos (2004), Leite and Dimitrakopoulos (2007) and Boland et al. (2008).

In this paper we study how conditional simulation can be used to incorporate explicit risk control to open pit mine design and evaluation. As a first step in this direction we propose a probabilistically constrained version of the ultimate pit problem that directly incorporates risk control into the optimization process. We illustrate this approach using data from a copper mine in Chile and describe the potential advantages and challenges it presents. In particular, we study the effect of varying the number of conditional simulations on the obtained results.

The rest of this paper is organized as follows. In Section 2 we review the standard ultimate pit problem and in Section 3 we introduce a probabilistically constrained version of this problem. In Section 4 we apply the probabilistically constrained ultimate pit problem and compare the results to other approaches such as generating the ultimate pit for each simulation and the hybrid pit approach introduced in Whittle and Bozorgebrahimi (2007). In Section 5 we study the effect or varying the number of conditional simulations on the results obtained by the probabilistically constrained ultimate pit problem. Finally, in Section 6 we give some conclusions and future research.

The Ultimate Pit Problem

The uncapacitated open-pit mine planning problem (U-PIT), also known as the ultimate pit problem, or optimal contour problem, consists in finding the set of blocks which would be extracted in the absence of capacity constraints. This problem is considered in a first stage of the life of mine production scheduling and is usually solved by the Lerchs and Grossmann algorithm (Lerchs and Grossmann, 1965). The U-PIT problem can be formulated as mathematical programming problem in the following way.

Let each block of a block-model of the studied mine be numbered from 1 to B and let $\mathcal{P} \subseteq \{1,\ldots,B\} \times \{1,\ldots,B\}$ be the set of immediate precedences in the mine. That is, if $(a,b) \in \mathcal{P}$ then a must be extracted before b. We say that a set of blocks $U \subseteq \{1,\ldots,B\}$ defines a *pit* if every block in U is such that all of its predecessors are also in U. That is, if $b \in P$ and $(a,b) \in \mathcal{P}$ then $a \in U$. With this the U-PIT problem can be formulated as the Integer Programming (IP) problem given by

$$\max \sum_{b=1}^{B} p_b x_b \tag{1}$$

s.t.

$$x_b \le x_a \quad \forall (a,b) \in \mathcal{P}, \tag{2}$$

$$x_b \in \{0,1\} \quad \forall b \in \{1,\ldots,B\}, \tag{3}$$

where p is the B-dimensional vector of profits obtained for extracting each block.

The U-PIT problem is usually incorporated into the planing process through the nested pit implementation of the Lerchs and Grossmann (see Whittle (1999)), which generates a sequence of optimal solutions to (1)–(3) for varying objective coefficient vector p. However, a single U-PIT problem can also provide important insight into a mining project. For an appropriate choice of objective coefficients, the contour of the ultimate pit generated by an optimal solution to (1)–(3) is a good estimate of the shape of the mine at the end of its life (Caccetta (2007)). For this reason, the objective value of the ultimate pit is commonly used as a rough estimate for the final value of a mine.

A Probabilistically Constrained Ultimate Pit Problem

An issue with using problem (1)–(3) to estimate the profitability of a mining project is that it only considers a single orebody model, which can be very different from the actual mineralization in the ground. To mitigate this problem Dimitrakopoulos et al. (2007) estimate the actual value of the obtained pits by recalculating their monetary value using several orebody models obtained through conditional simulation. Another approach is introduced in Whittle and Bozorgebrahimi (2007) where set-theory is used to generate so-called *Hybrid Pits* that are generated by studying the ultimate pits for several conditional simulations at a time. Both approaches provide significant improvements over the deterministic U-PIT problem applied to a single orebody model. However, in both cases the evaluation or generation of solutions is only done after a series of U-PIT problems are solved. To directly risk control directly into the optimization process of the U-PIT problem we propose adding a probabilistic constraint to problem (1)–(3) by using an approach introduced in Kataoka (1963).

Assume that through conditional simulation we have obtained n equally probable orebody models for the mine. For each $i \in \{1,\ldots,n\}$ let p^i be the B-dimensional block-profit vector according to orebody model i and let $\tilde{p}$ be a B-dimensional random vector such that $\tilde{p} = p^i$ with equal probability for each $i \in \{1,\ldots,n\}$. A possible way to incorporate this information into the U-PIT problem is to calculate the expected monetary value of each pit given by a solution x feasible for (2)–(3). However, this approach does not incorporate any consideration for risk so we propose using the value

$$v_\delta(x) := \max\left\{ z : \mathbb{P}\left(\sum_{b=1}^{B} \tilde{p}_b x_b \geq z\right) \geq \delta \right\} \tag{4}$$

for a given confidence level $\delta \in (0,1)$. This value can be considered as a robust version of objective (1) because it assures that a value equal or greater than $v(x)$ is achieved for solution x with sufficiently high probability δ over random vector $\tilde{p}$. Then, to study the potential monetary value and variability of the mine we can consider the problem

$$V_\delta := \max\{v(x) : (2)\text{–}(3)\} \tag{5}$$

and plot V_δ for appropriate values of δ.

Because $\tilde{p}$ has a finite distribution we can obtain V_δ by solving the equivalent deterministic IP given by

$$
\begin{aligned}
V_\delta := \max \quad & z && && (6)\\
s.t. \quad & && && (7)\\
& \sum_{b=1}^{B} \tilde{p}_b x_b \geq z - M_s y_i && \forall i \in \{1,\ldots,n\} && (8)\\
& x_b \leq x_a && \forall (a,b) \in \mathcal{P}, && (9)\\
& x_b \in \{0,1\} && \forall b \in \{1,\ldots,B\}. && (10)\\
& \sum_{i=1}^{n} y_s \leq \lfloor (1-\delta)n \rfloor && && (11)\\
& y_s \in \{0,1\} && \forall i \in \{1,\ldots,n\}, && (12)
\end{aligned}
$$

for sufficiently large values M_s.

Table 1: Instance characteristics

Characteristic	Value
Block size	$10 \times 10 \times 12$
Number of blocks	34 140
Copper price	6.6 \$/kg
Selling cost	0.6 \$/kg
Mining recovery	95%
Processing recovery	95%
Mining cost	1.8 \$/tonne
Processing cost	5.5 \$/tonne

Application to a Copper Mine

We test the approach proposed in Section 3 on a block model corresponding to a section of the Andina copper mine in Chile. The characteristic of this instance are included in Table 1.

For this instance we generated 10 orebody models using the conditional simulation software TBSIM (Emery and Lantuejoul, 2006). Let p^i be the 34140-dimensional block-profit vector generated with the i-th simulation for $i \in \{1,\dots,10\}$.

Using these 10 models we calculate $v_\delta(x)$ for pits x obtained by different methods and for $\delta \in \{0.8, 0.9, 1.0\}$. The methods used are the following.

Average Let x be the optimal solution to problem (1)–(3) with $p_b = \frac{1}{10}\sum_{i=1}^{10} p_b^i$ for each block b. This is similar to solving the U-PIT problem for p obtained through ordinary kriging.

Simulations For each simulation $i \in \{1,\dots,10\}$ let x^i be the optimal solution to problem (1)–(3) with $p_b = p_b^i$ for each block b. For each $\delta \in \{0.8, 0.9, 1.0\}$ let x be the x^i that maximizes $v_\delta(x)$. This approach is similar to the one used in Dimitrakopoulos et al. (2007).

Hybrid For each simulation $i \in \{1,\dots,10\}$ let x^i be the optimal solution to problem (1)–(3) with $p_b = p_b^i$ for each block b. Then use these x^i to calculate the 10 possible hybrid pits as described in Whittle and Bozorgebrahimi (2007). For each $\delta \in \{0.8, 0.9, 1.0\}$ let x be the hybrid pit that maximizes $v_\delta(x)$.

SIP For each $\delta \in \{0.8, 0.9, 1.0\}$ let x be the optimal solution to (7)–(12).

All instances of problem (1)–(3) were solved using the push-relabel algorithm for max-flow implemented in the C library EGLIB that is available at `http://www.dii.uchile.cl/~daespino/EGlib_doc/main.html`. Problem (7)–(12) was solved using the commercial IP software ILOG CPLEX v11 (ILOG, 2008) and all experiments were conducted on a 3.0Ghz Pentium 4 workstation with 1.5Gb of RAM.

Figure 1 shows the results for these experiments. As expected we can see that the worst solutions are obtained with Average, that Simulations and Hybrid can provide some improvement and that SIP presents the best results. However, the limited number of simulations only provides a narrow view of the potential monetary value of the mine and its variability.

Effect of the Number of Conditional Simulations

We now study how the number of simulations can affect the obtained results. For this we generated 90 additional orebody models for a total of 100 models.Our first experiment is to use the 100 simulations to

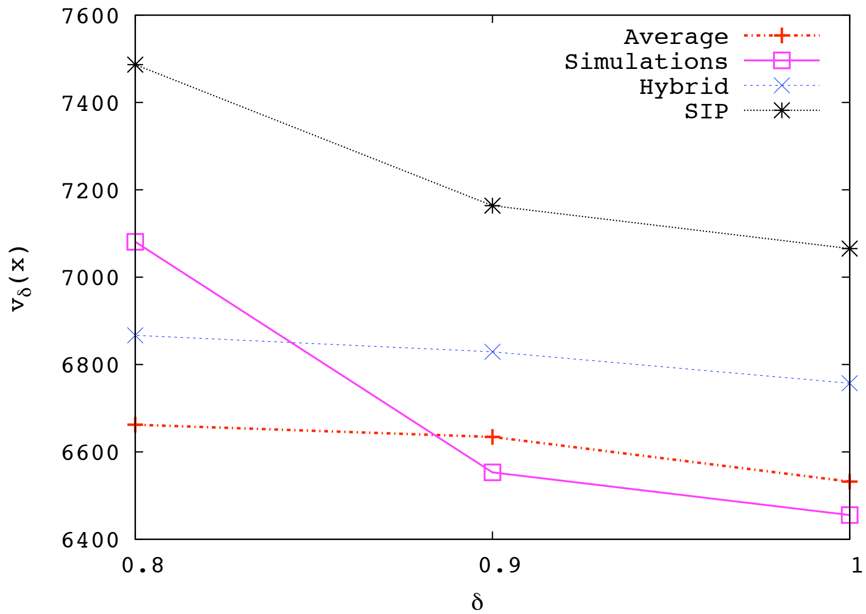

Figure 1: Results for 10 simulations.

re-evaluate $v_\delta(x)$ for all solutions obtained in the previous section for $\delta \in [0.8, 1]$ in increments of 0.01. For each δ and for each method we again selected the solution that provided the largest value of $v_\delta(x)$. Figure 2 shows the results for this experiment.

We can see that the results have dramatically changed. The relative quality of the solutions obtained by the four methods remains similar to the previous experiment, with SIP and Average usually obtaining the best and worst solutions respectively. However, all values have been significantly decreased, particularly for $\delta = 1$ in which $v_\delta(x)$ is forced to be greater than $\sum_{b=1}^{B} p_b^i x_b$ for every one of the 100 orebody models. This decrease could be caused by the myopic behaviour of the methods in the previous section, which could only use the first 10 simulations. For this reason, for our final experiment we repeat the experiment from the previous section using all 100 simulations from the start. The results for this experiment are presented in Figure 3. We note however, that this time problem (7)–(12) was significantly harder to solve, so the results for SIP are not the optimal solution of (7)–(12), but the best solution found within 12 hours of computing time.

We can now see a clear ordering in the relative quality of the solutions obtained by the four methods, with SIP again providing solutions that are up to 13.5% better than the ones obtained by the other methods. However, the values obtained by Simulations and Hybrid were, in average, only 6.8% and 7.3% smaller than the ones obtained by SIP. This shows that Simulations and Hybrid can be very good heuristics for solving (7)–(12), which is extremely interesting considering that solution times for Simulations and Hybrid were a few minutes compared to the 12 hours used by SIP. We can also see that, although the decrease in values from Figure 1 to Figure 2 has been slightly mitigated in Figure 3, we still have a difference of up to 28.6% between the values predicted with 10 and 100 simulations.

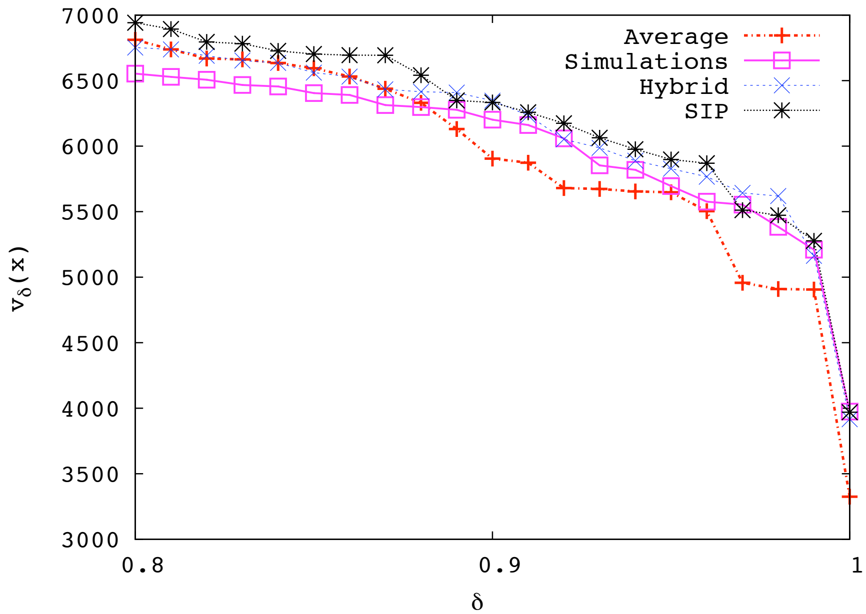

Figure 2: Results for 10 simulations evaluated using 100 simulations.

Conclusions and Further Work

We have proposed a probabilistically constrained version of the ultimate problem as a more robust way of estimating the monetary value of a mine. Preliminary results show that solving this problem directly can provide better solutions than previous approaches. Unfortunately, when the number of simulations increases the computational burden of this problem can become quite severe. However, good heuristics for this problem can be obtained by using ideas from Dimitrakopoulos et al. (2007) and Whittle and Bozorgebrahimi (2007). The use of these heuristics together with techniques similar to the ones in Luedtke et al. (2008) and Kucukyavuz (2009) could lead to efficient solution methods for this problem.

We have also seen that the obtained results can be significantly sensitive to changes in the number of conditional simulations considered. This phenomenon definitely requires additional study and care should be taken when deriving conclusions from experiments with a limited number of simulations. A potential solution to this issue lies on the study of sample average approximation of probabilistically constrained problems (Luedtke and Ahmed, 2008).

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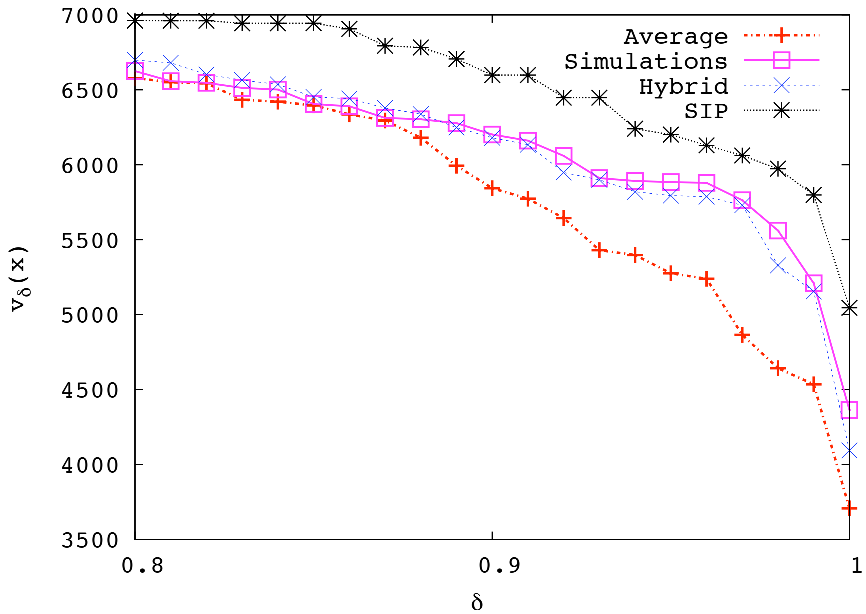

Figure 3: Results for 100 simulations.

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